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ADAPTIVNÍ PREDICKE PŘECHODŮ MEZI LOKÁLNÍMI ATRAKTORY LORENZOVA SYSTÉMU

Adaptive Prediction of Transitions between Local Attractors of Lorenz's System

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Abstract – The paper demonstrates an adaptive prediction of trajectory transitions between local basins of attraction of deterministic chaotic systems using low-dimensional dynamic quadratic neural unit with periodically forced neural inputs. The forthcoming transitions of higher dimensional chaotic systems are predicted by a low dimensional discrete dynamic neural unit implemented as a special adaptive forced oscillator. The real-time sample-by-sample evaluation of complex system behavior is based on monitoring of parameters of an adaptive model during its adaptation. The behavior of chaotic systems in the state-space is adaptively transformed to system behavior in an approximated parameter-space using special higher order nonlinear neural unit forced with periodical inputs. Added forcing inputs naturally allow a low dimensional dynamic neural unit to better approximate higher dimensional behavior; the forcing neural inputs are initially configured upon analysis of frequency spectra of the evaluated time series.

Key words: adaptive evaluation, chaos, inter-attractor transition, higher order nonlinear neural unit, forced nonlinear dynamic oscillator

1. Introduction

The Lorenz's system [1] is one of the most familiar chaotic systems. Most, it is known as a continuous nonlinear dynamic non-dissipative system that exhibits chaos in the three dimensional state-space (Eq.(1)). Typical for the complex trajectory of Lorenz's system are trajectory transitions between local basins of attraction thus creating the well known two-wing shape of the whole system attractor (Fig.1). The simulations experiments in this work focuses on the detection of the forthcoming transitions between the local basins of attractions. The contribution of this work is that the local transitions of three-dimensional nonlinear dynamic system (Lorenz's system) are adaptively predicted by a low dimensional dynamic quadratic neural unit (QNU) with forcing inputs. The notion of quadratic and other nonconventional neural units was originally introduced in [3] and more recently developed also in works [4] and [5]. In this paper, the dynamics of the QNU is of a first order (the unit performs in 1-D state space) and the unit is further enhanced by appropriate forcing inputs so it creates a forced dynamic oscillator. It is shown in [2], that adding forcing inputs to a dynamic system naturally increases dimensionality of a unit by an additional dimension and thus it allows better approximation of more complex behavior. The adaptive approach for evaluation of high dimensional complex system by lower dimensional neural units with forcing inputs was recently introduced in [5], where the forcing inputs were originally designed with respect to the physiological nature and the spectral frequency analysis of the evaluated signals, i.e. heart beat tachograms. An original technique for visualization of approximated system in its parameter space was also introduced in [5].

$$\begin{aligned} \dot{x}(t) &= 10y(t) - 10x(t) \\ \dot{y}(t) &= 28x(t) - y(t) - x(t)z(t) \\ \dot{z}(t) &= x(t)y(t) - \frac{8}{3}z(t) \end{aligned} \tag{1}$$

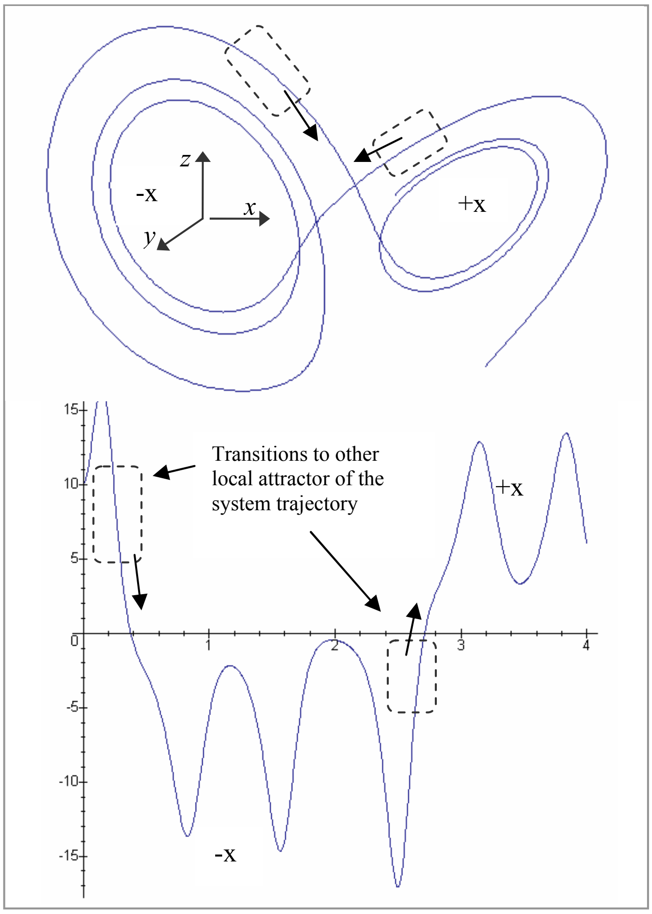

Fig. 1: The evaluated Lorenz's attractor (top). The goal is to detect incoming transitions between the local attractors of the system trajectory.

In this paper, 1-D static and dynamic quadratic neural units with forcing inputs (thus better approximating higher dimensional systems) are shown as designed for prediction of local attractor transitions of a Lorenz's system running in chaotic regime. The adaptation plot (introduced as monitor plot in [5]) that visualizes the real-time behavior of a neural unit in its parameter space is introduced with focus on vertical patterns detection of incoming significant changes in system behavior (the inter-attractor transition). The results of forced one-dimensional QNU for early detection of incoming inter-attractor transients of (higher dimensional) Lorenz's system running in chaotic mode are shown via adaptive monitoring of state variable $x(t)$ from system Eq.(1).

2. Quadratic Neural Units with Forcing Inputs

The quadratic neural unit with linear somatic operation and two forcing neural inputs is shown in Eq.(2). In case of static QNU, the neural input $xa_3=x(k)$ is fed to a unit as an external input (previously obtained from system in Eq.(1)). In case of dynamic QNU, the neural output $x(k+1)$ is delayed by a discrete sampling period and fed back to neural input thus creating a recurrent neural unit ΔT.

$$
\begin{aligned}
& x(k+1)=\sum_{i=0}^{3}\sum_{j=i}^{3} w_{ij} xa_i(k) xa_j(k), \\
& where \quad w_{00}=0,\ xa_0=1 \\
& u_1 = xa_1 = \cos(\omega_1 \cdot \Delta T + \phi_1), \\
& u_2 = xa_2 = \cos(\omega_2 \cdot \Delta T + \phi_2), \\
& xa_3 = x(k).
\end{aligned}
\tag{2}
$$

The frequencies ω_1 and ω_2 were chosen as the two most significant frequencies of frequency spectra of state variable $x(t)$, the phase delays φ_1 and φ_2 of forcing inputs u_1 and u_2 of QNU in Eq.(2) were configured manually to appropriately fit the plot of variable $x(t)$ from Eq.(1) without the use of any sophisticated routine. Note that amplitudes of both neural inputs are included in the neural weights and were configured automatically later during the adaptation of a unit. According to the technique for stable adaptation of neural units introduced in [5], the neural weights w_{ij} of static QNU (Fig.1.) were first adapted so the static unit approximated state variable $x(t)$ of system Eq.(1). Then, the neural weights w_{ij} of the static neural unit were used as initial weights of the dynamic QNU to assure its stability during the adaptation (for an appropriate learning rate that may be a very small number). Both the static and dynamic neural units were adapted by the gradient-descent based back-propagation learning rule [3], and the initial use of static neural unit prior the dynamic neural unit avoids the issues of its adaptation instability. Consequently, the dynamic QNU shown in Eq.(2) is used for the higher accuracy of system approximation naturally resulting from its recurrent delay feedback of $x(k)$.

3. The Adaptation Plot

The plot of markers detecting the unusually large neural weight increments is called as the Adaptation Plot (formerly the monitor plot in [5]).

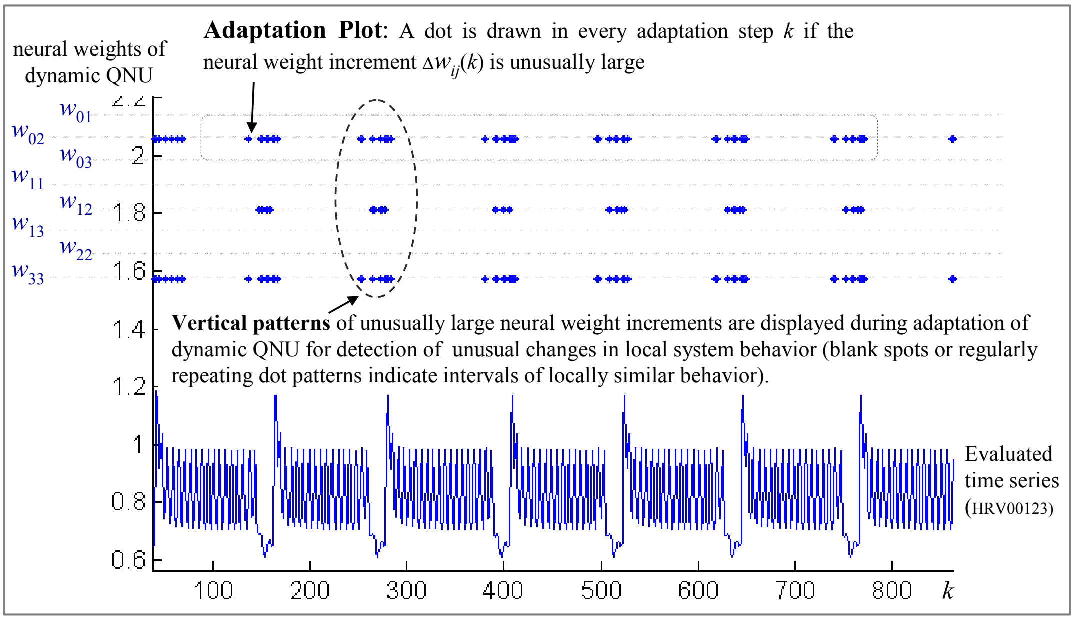

Fig. 2: The adaptation plot for monitoring and visualization of unusual neural weight increments during the adaptation of dynamic quadratic neural unit.

The algorithm for drawing the markers of each unusually large neural weight increments in the Adaptation Plot is as follows:

IF ABS(w_{ij} $(k+1)$- $w_{ij}(k-1)$) $> p \cdot$ Ref_Δw_{ij} THEN detection is positive, record and draw the marker ;

where p is the detection sensitivity parameter, w_{ij} represents adaptable neural weights of QNU, n is the number of all adaptable neural parameters including the optional signal input preprocessor, N is the number of samples. For more details on the Adaptation Plot, we refer kind readers to [5].

4. The results

The quadratic neural units were adapted to visualize the unusual weight increments during the adaptation of dynamic QNU Eq.(2) with sampling ΔT=0.01 seconds during approximation of state variable $x(t)$ of Lorenz's system in Eq.(1). The appropriate detection sensitivity parameters p were found so the markers appearing in top right corners in Fig.4 and Fig.5 clearly predicted forthcoming local attractor transition (see the insets in Fig.4 and Fig.5). The number of adaptation passes for static QNU was five and the dynamic QNU was consequently adapted once to visualize the neural increments in the adaptation plot.

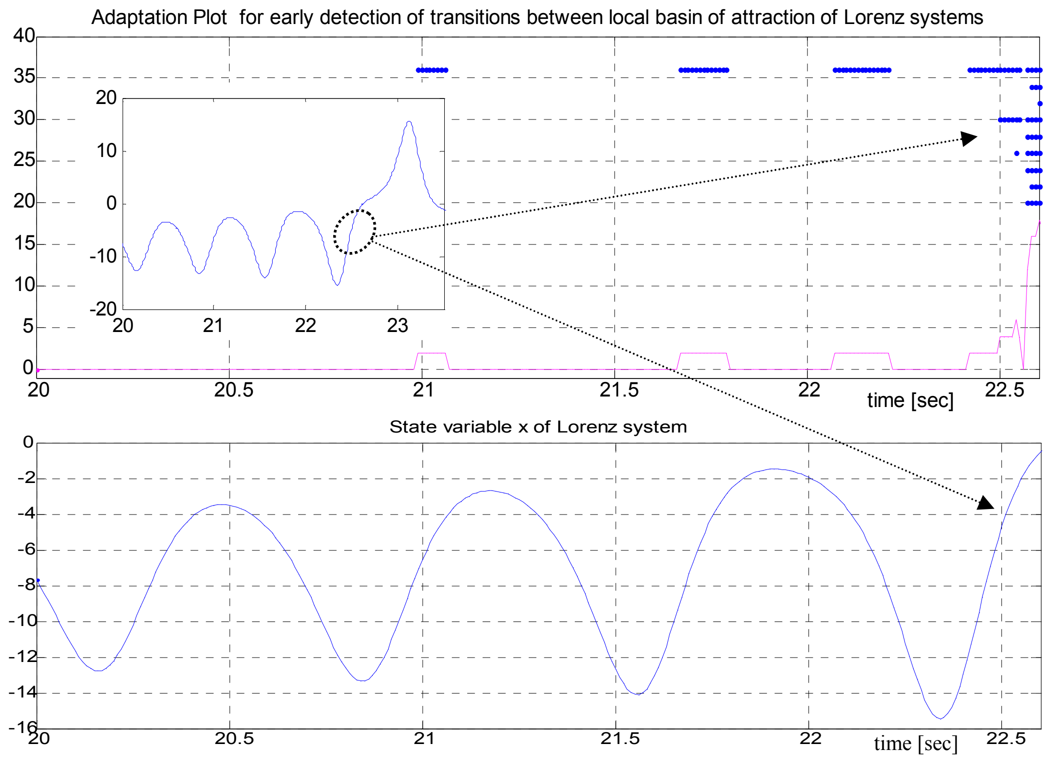

Fig. 4: Prediction of the transient of variable *x* from region of negative values (-local attractor) to the region of positive values (+local attractor). Notice the blue markers appearing in top right corner that indicate the incoming transition to other local attractor.

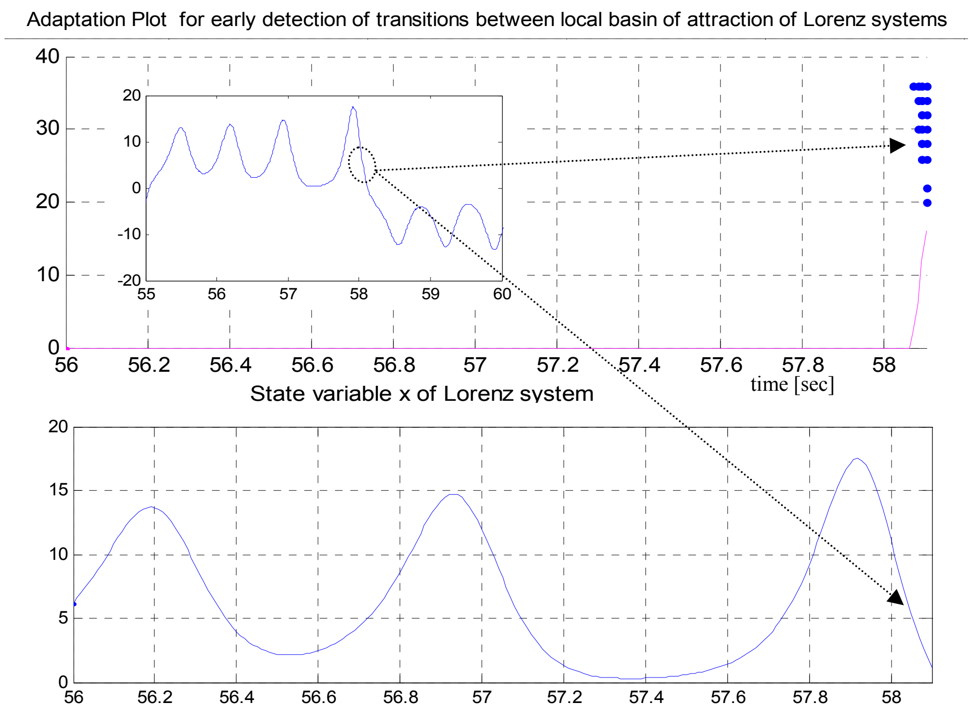

Fig. 5: Prediction of the transient of variable *x* from region of negative values (-local attractor) to the region of positive values (+local attractor).

5. Conclusions

The results have confirmed that incoming transitions between local attractors of three-dimensional Lorenz's system running in highly chaotic regime can be well ahead predicted by the adaptive approach with the use of one-dimensional (single-recurrence) quadratic neural unit neural unit enhanced by the appropriate forcing inputs that naturally improves their approximating capability. The dynamic QNU with forced inputs was able to detect incoming transitions of state trajectory of usually at least 10 samples ahead, that is in fact much higher number than it is its own dimensionality (the single recurrence QNU was used). The research work and simulation experiments in this paper have come along our emerging research on adaptive evaluation of complex systems merging the use of new cognitive systems such as the nonconventional neural architectures [4] and chaos theory in order to handle complex behavior of real, esp., biological systems [5].

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