

Squeezing flow of compressible viscoelastic material with partial slip

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Abstract. This contribution deals with the problem of squeezing flow of compressible viscoelastic material considering partial slip at the surface of compressing circular plates. Rheological behavior of compressed samples is described by a power law model of shear deformation (consistency K and flow behavior index m) and viscoelastic properties are expressed using power law relation for first normal stress difference (K' , m'). Alternative formulation takes into account only elongation and normal stresses. Squeezed material is considered as a foam, i.e. incompressible continuous phase filled by air bubbles. Partial slip at wall is expressed by dimensionless parameter δ proportional to slip velocity that is independent of consistency and stresses. Suggested mathematical models result to analytical expression for axial force as a function of compression rate and the thickness of gap. The models are applied for experiment with the sample of collagen compressed between two parallel disks of the texture analyzer instrument TA-XT2i.

INTRODUCTION

Quasi-static compression flow of tested material between two approaching plates (parallel discs) is called squeezing flow. Rheological properties are evaluated from relationships between the normal force acting on a moving plate, thickness of closing gap and velocity of moving plate. This flow was analyzed for different conditions and liquids, and a review of existing solutions is given in the references Steffe' (1996), Barbosa Canovas (1998). Historically the first analysis of the squeezing flow was presented by Stephan (1874) for Newtonian liquids, followed by Scott (1929) for Bingham liquids, and Scott (1935) for power law and Herschel Bulkley liquids. Yield stress controversies were later on resolved by Covey & Stanmore (1981). Their results can be summarized to the prediction of axial force as a function of gap between plates

$$F = \frac{A}{h^m} + \frac{B}{h} \quad (1)$$

assuming a constant velocity of moving plate. The exponent m varies typically in the range 2 (prevailing effect of yielding) to 3 (prevailing viscous effects). The second term disappears for liquids with zero yield stress (in fact the coefficient B is directly proportional to τ_y).

All these works calculate the axial force F by integrating pressure on the surface of plates. Viscoelastic model Barhon (1993) adds to the pressure the axial viscous stresses, based upon power law approximation of the first normal stress difference (radial minus axial normal stresses), giving

$$F = \frac{A}{h^m} + \frac{B}{h^{2q}} \quad (2)$$

where the first term describes the contribution of pressure (exponent m varies again within the range from 2 to 3) and the second term represents first normal stress differences expressed as a function of shear rate. The exponent q varies from zero (mayonnaise) up to 0.5 (butter)

according to experimental data presented by Steffe (1996). This model neglects elongation and therefore the case, when an ideally lubricated cylindrical sample that slides at surface of plates and preserves its ideal cylindrical shape, cannot be described correctly (zero shear rate means uniform atmospheric pressure and zero difference of normal stresses). This restriction was eliminated by Laun (1999), who calculated axial normal stresses directly from the elongation rate determined by velocity of approaching plates (axial normal stress was approximated by power law function of elongation). Result for perfectly lubricated plates has a simple form

$$F = \frac{A}{h^m} \quad (3)$$

Only few experimental results concerning the value of exponent m are available, preliminary experiments with collagen, Landfeld (2015), indicate rather low value $m \sim 0.5$.

Squeezing flow of viscoelastic liquids described by upper convected Maxwell constitutive equation was presented by Phan-Thien and Tanner (1983). The perturbation solution is rather complicated but results can be simplified to

$$F = \frac{A}{h^3} + \frac{B}{h^4} \quad (4)$$

where the parameter A depends not only upon the velocity but also upon the acceleration of plates. Squeezing flow of solid-like materials modelled by the Kelvin Meyer Voigt equation was analyzed by Phan-Thien (2000) numerically.

According to our best knowledge no models taking into consideration compressibility of the squeezed material have been realized. This is a motivation for the present article that extends the Barhon's approach by compressibility and partial wall slip. The developed method is applied for description of squeezing flow experiments with highly concentrated collagen solution.

THEORY OF SQUEEZING FLOW DOMINATED BY SHEAR

In our case the inertial and gravitational forces are neglected. We consider cylindrical coordinate system and geometry given in Fig. 1.

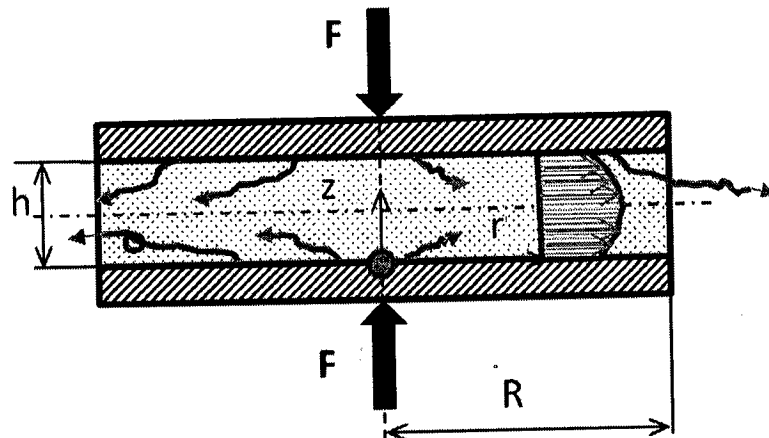


Fig. 1 Scheme of the squeezing flow geometry and parameters (visualization of polymer fibers)

We also consider radial velocity dependent on radius r and position z : $u_r = u_r(r, z)$. Isotropic pressure p inside the compressed sample depends only on radial distance $p = p(r)$. It is assumed that both components of axial force (contribution of isotropic pressure and contribution of first

normal stress difference) depend only upon the shear rate. Effect of the elongation flow is neglected and therefore the method cannot be applied for perfectly lubricated plates. Momentum balance in the radial direction can be expressed by the equation

$$\frac{\partial}{\partial r}(\tau_{rr} - p) = -\frac{\partial \tau_{zr}}{\partial z} - \frac{\tau_{rr} - \tau_{zz}}{r} \quad (5)$$

assuming $\tau_{\phi\phi} \approx \tau_{zz}$ (therefore assuming that the second normal stress difference is zero). This equation enables to calculate pressure and the resulting force acting upon a moving circular disc of the radius R (from one side acts atmospheric pressure, from the other side pressure and viscous stress)

$$\begin{aligned} F &= 2\pi \int_0^R (\tau_{zz} - p)|_{z=h} r dr - \pi R^2 p_a \\ &= 2\pi \int_0^R (\tau_{rr} - p + \tau_{zz} - \tau_{rr})|_{z=h} r dr - \pi R^2 p_a \end{aligned} \quad (6)$$

The first term in the integrand can be integrated per partes and the second term can be replaced by the first normal stress difference $N_1 = \tau_{rr} - \tau_{zz}$ giving after application of Eq. (5) and little manipulation

$$F = \pi \int_0^R \left(r^2 \frac{\partial \tau_{zr}}{\partial z} - r N_1 \right) |_{z=h} dr \quad (7)$$

What follows is only an approximation based upon assumption that the shear stress τ_{zr} as well as N_1 depend only upon the shear rate. Shear stress and first normal stress difference can be described by power laws

$$\tau_{rz} \cong K \left(\frac{\partial u_r}{\partial z} \right)^m \quad (8)$$

$$N_1 \cong K' \left(\frac{\partial u_r}{\partial z} \right)^{m'} \quad (9)$$

The momentum balance (5) can be simplified by canceling terms with normal stresses (this is exact for Newtonian liquids, for power law liquids it is only an approximation)

$$0 = \frac{1}{r} \frac{\partial}{\partial r} (r \tau_{rr}) - \frac{\tau_{\phi\phi}}{r} + \frac{\partial \tau_{zr}}{\partial z} - \frac{\partial p}{\partial r} \quad (10)$$

$\cong 0$

Then the constitutive equation (8) in combination with the momentum balance (10) give the following relationship between the shear rate and the gradient of pressure.

$$\frac{\partial u_r}{\partial z} = \left(-\frac{1}{K} \frac{\partial p}{\partial r} \Big|_{z-h} - \frac{h}{2} \right)^{\frac{1}{m}} \quad (11)$$

Combining (11), (8), (9) and substituting into (7) we arrive to the final equation for axial force, expressed only by means of gradient of pressure dp/dr

$$F = \pi \int_0^R r^2 \frac{\partial p}{\partial r} dr - \pi K' \left(\frac{h}{2K} \right)^{\frac{m'}{m}} \int_0^R r \left(\frac{\partial p}{\partial r} \right)^{\frac{m'}{m}} dr \quad (12)$$

Remark: It is obvious that the sign of both terms is different, the first viscous term increases the load on the moving discs, while the second viscoelastic term mitigates the load. Possible explanation is shown in Fig.1 illustrating polymeric fibers attached by one end to plate and stretched by flow at the other end. Stretched fibers exert opposite forces than the pressure forces. There should be applied a restriction to the magnitude of the second term and to the value of consistency K' because it is hardly believable that when a critical velocity will be exceeded a spontaneous suction begins and that it would be necessary to brake the motion.

The expression (12) is independent of wall slip and this effect is included in the boundary condition for the radial velocity profile, obtained by integration of (11)

$$u_r = \frac{ru_R}{R} + \frac{m}{m+1} \left(-\frac{1}{K} \frac{\partial p}{\partial r} \right)^{\frac{1}{m}} \left(\left| \frac{h}{2} \right|^{\frac{m+1}{m}} - \left| \frac{h}{2} - z \right|^{\frac{m+1}{m}} \right) \quad (13)$$

where u_R is slip velocity at the rim of circular plate (slip velocity increases linearly with the increasing radius). The dimensionless slip parameter δ was introduced by Laun (1999)

$$\delta = \frac{hu_R}{-\frac{dh}{dt}R} \quad (14)$$

Value $\delta=0$ represents no wall slip (zero radial velocity at $z=h$), while $\delta=0.5$ describes perfect lubrication, therefore uniform radial expansion of sample with radial velocity u_r independent of z (unidirectional compression).

Radial pressure profile $p(r)$ follows from the mass balance (stating that the rate of change of mass in a cylinder of radius r compressed by approaching circular plates must be the same as the mass flowrate calculated from the radial velocity profile (13))

$$-\pi r^2 \frac{dh}{dt} = 2\pi r \left(h \frac{ru_R}{R} + \frac{2m}{2m+1} \left(-\frac{1}{K} \frac{\partial p}{\partial r} \right)^{\frac{1}{m}} \left(\frac{h}{2} \right)^{\frac{2m+1}{m}} \right) \quad (15)$$

after a little rearrangement

$$\left(-r \frac{dh}{dt} (1-2\delta) \frac{2m+1}{4m} \right)^m \left(\frac{2}{h} \right)^{2m+1} K = -\frac{\partial p}{\partial r} \quad (16)$$

and after integration

$$p(r) = Z(R^{m+1} - r^{m+1}) + p_a \quad (17)$$

$$\frac{\partial p}{\partial r} = -(m+1)Zr^m \quad (18)$$

where Z represents compression rate and can be evaluated from the given velocity dh/dt , thickness of gap h and the constitutive parameters K, m

$$Z = \frac{K}{m+1} \left(\frac{2}{h} \right)^{2m+1} \left(-\frac{2m+1}{4m} \frac{dh}{dt} (1-2\delta) \right)^m \quad (19)$$

Resulting force follows from Eq. (12) substituting gradient of pressure (18)

$$F = \pi Z \frac{m+1}{m+3} R^{m+3} - \pi \left(Z(m+1) \frac{h}{2K} \right)^{m'/m} \frac{K'}{m'+2} R^{m'+2} \quad (20)$$

The relationship (20) takes into account a partial wall slip, predicting for example zero force for perfect slip $\delta=0.5$. This result can be applied only for incompressible fluids, therefore for

the situation when the mass flowrate of the expressed material equals $\pi R \rho_l dh/dt$, while the actual mass flowrate of compressible material is lower. The analysis of compressible fluids begins with the calculation of the volume of foam containing the mass of air M_g and the mass M_l of incompressible phase having density ρ_l

$$V = \frac{M_l}{\rho_l} + \frac{M_g}{\rho_g} = M_l \left(\frac{1}{\rho_l} + \frac{\omega_g}{\rho_g} \right) = M_l \left(\frac{1}{\rho_l} + \frac{\omega_g R_g T}{p} \right) = \frac{M_l}{\rho_l} \left(1 + \frac{\Omega}{p} \right) \quad (21)$$

The ratio Ω/p represents volumetric fraction of bubbles at temperature T . Typical values of $\Omega = \omega_g R_g T \rho_l$ are within $10^4 - 10^5$ Pa and are zero for incompressible liquids. The mass flowrate of porous material containing gas bubbles corresponding to the radial pressure profile $p(r)$ is expressed by the following integral

$$\dot{m} = -\frac{dh}{dt} \rho_l \int_0^R \frac{2\pi r dr}{1 + \frac{\Omega}{p(r)}} = -\frac{dh'}{dt} \rho_l \pi R^2 \quad (22)$$

The equation (22) introduces the corrected velocity (dh'/dt) giving the same mass flowrate as in the case of incompressible fluid. The ratio of reduced and actual velocity is a scale factor φ

$$\varphi = \frac{dh'/dt}{dh/dt} = \frac{2}{R^2} \int_0^R \frac{r dr}{1 + \frac{\Omega}{p(r)}} = \frac{2}{R^2} \int_0^R \frac{r dr}{1 + \frac{\Omega}{Z(R^{m+1} - r^{m+1}) + p_a}} \quad (23)$$

The integral (23) cannot be integrated analytically but can be approximated by linearization of integrand, giving

$$\varphi \cong \frac{ZR^{m+1} + p_a}{ZR^{m+1} + p_a + \Omega} \quad (24)$$

The scale factor (24) is always less than one, decreases with the increasing mass fraction of bubbles and is approaching to unity for infinitely small gap (therefore for infinitely large pressure, when the volumetric fraction of gas is zero and material becomes practically incompressible). And this is the essence of the suggested method: Given the gap h and velocity dh/dt the scale factor φ is calculated according to Eq. (24). The reduced velocity dh'/dt is used for calculation of Z in Eq. (19) and the force F is calculated according to Eq. (20). It should be stressed that the method is quite heuristic and is not based upon basic principles (for example the procedure cannot be iterated by repeated evaluation of new $Z(\varphi)$ because the φ would be monotonically decreasing). The hypothesis can be described in words as the assumption that the radial flow resistance and the corresponding radial pressure profile depends only upon the mass flowrate. This mass flowrate can be calculated from the volumetric flowrate determined by the velocity of plates and by density of foam that depends upon pressure.

THEORY OF SQUEEZING FLOW DOMINATED BY ELONGATION

A quite opposite extreme is represented by pure elongation, the case when the radial velocity is independent of z , the case when the compressed cylindrical sample slides without friction on the surface of discs, the case without shear stresses. The elongation rates for

incompressible material are independent of radial and axial coordinates and depend only upon time

$$\dot{\epsilon}_r = \dot{\epsilon}_\phi = \frac{\partial u_r}{\partial r} = \frac{u_r}{r} = \frac{-1}{2h} \frac{dh}{dt} \quad (25)$$

$$\dot{\epsilon}_z = \frac{\partial u_z}{\partial z} = \frac{-1}{h} \frac{dh}{dt} \quad (26)$$

This follows from the linear relationship of radial and axial velocities upon r and z , $u_r = r\dot{\epsilon}_r$, $u_z = z\dot{\epsilon}_z$ and from the mass balance of expressed material. Stresses corresponding to the rate of elongation can be approximated by a power law relationship

$$\tau_{rr} = \tau_{\phi\phi} = K_e \left(\frac{-1}{2h} \frac{dh}{dt} \right)^n \quad (27)$$

$$\tau_{zz} = K_e \left(\frac{-1}{h} \frac{dh}{dt} \right)^n = 2^n \tau_{rr} \quad (28)$$

Momentum balance in the radial direction (5) reduces to the equation

$$\frac{\partial}{\partial r} (\tau_{rr} - p) = -\frac{\tau_{rr} - \tau_{\phi\phi}}{r} = 0 \quad (29)$$

that can be integrated with the boundary condition for total dynamic stress ($p - \tau_{rr}$) at the rim ($r=R$): $p = \tau_{rr} + p_a$. Resulting pressure is therefore a constant independent of radius (p can be calculated from (29) and (27)) and the overall force acting upon the moving disc is given by

$$F = 2\pi \int_0^R r(p - \tau_{zz}) dr - \pi R^2 p_a = \pi R^2 (\tau_{rr} - \tau_{zz}) = \pi R^2 K_e \left(\frac{-1}{2h} \frac{dh}{dt} \right)^n (1 - 2^n) \quad (30)$$

Compressibility is respected quite empirically as a correction of the disc velocity dh/dt

$$F = \pi R^2 K_e \left(\frac{-\phi dh}{2h dt} \right)^n (1 - 2^n) \quad (31)$$

$$\phi = \frac{p_a + \tau_{rr}}{p_a + \tau_{rr} + \Omega} \quad (32)$$

This is almost the same approach as in the previous paragraph.

NUMERICAL EXPERIMENTS

Relationships for the force F according to Eq. (20) (flow dominated by shear) and Eq. (31) (elongation flow) were implemented in a short MATLAB program. Results for the parameters presented in the table 1 are shown in Figs. 2, 3, 4, 5.

Table 1. Parameters of the model

R	dh/dt	K	m	K'	m'	p_a	δ	Ω
[m]	[m/s]	[Pa.s ^m]	[-]	[Pa.s ^{m'}]	[-]	[Pa]	[-]	[Pa]
0.025	0.001	100	0.6	10	0.5	10 ⁵	0.01	10 ⁵

compressibility correction

~~Complicated and lengthy~~

~~the model is rather~~

24

The following Fig.2 demonstrates relative effect of viscoelasticity. It is seen that the values $K'/K > 10$ therefore very high first normal stress differences are above the stability limit and predicts negative resultant forces.

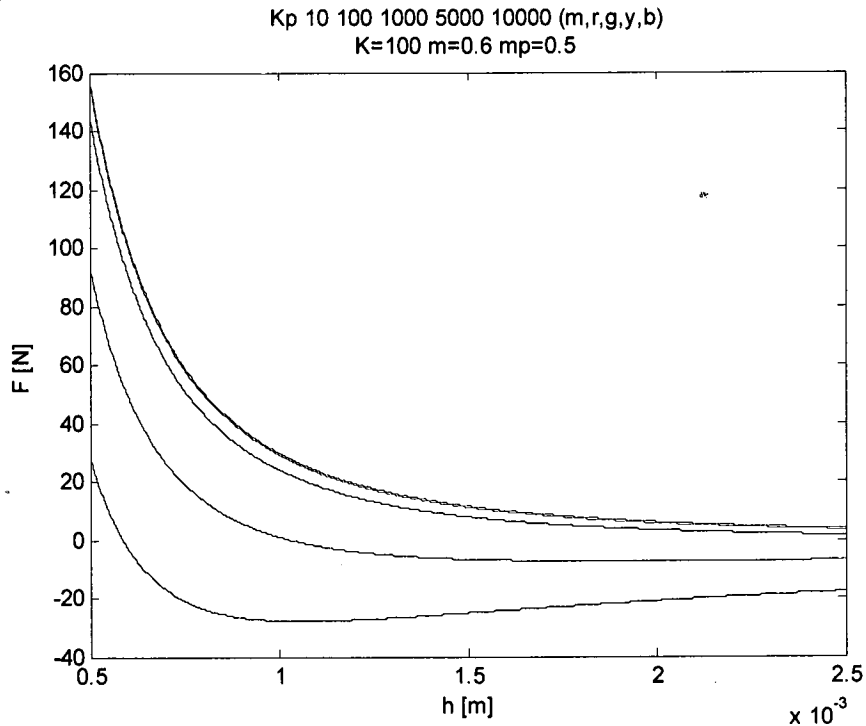


Fig.2 Force for different ratio K'/K

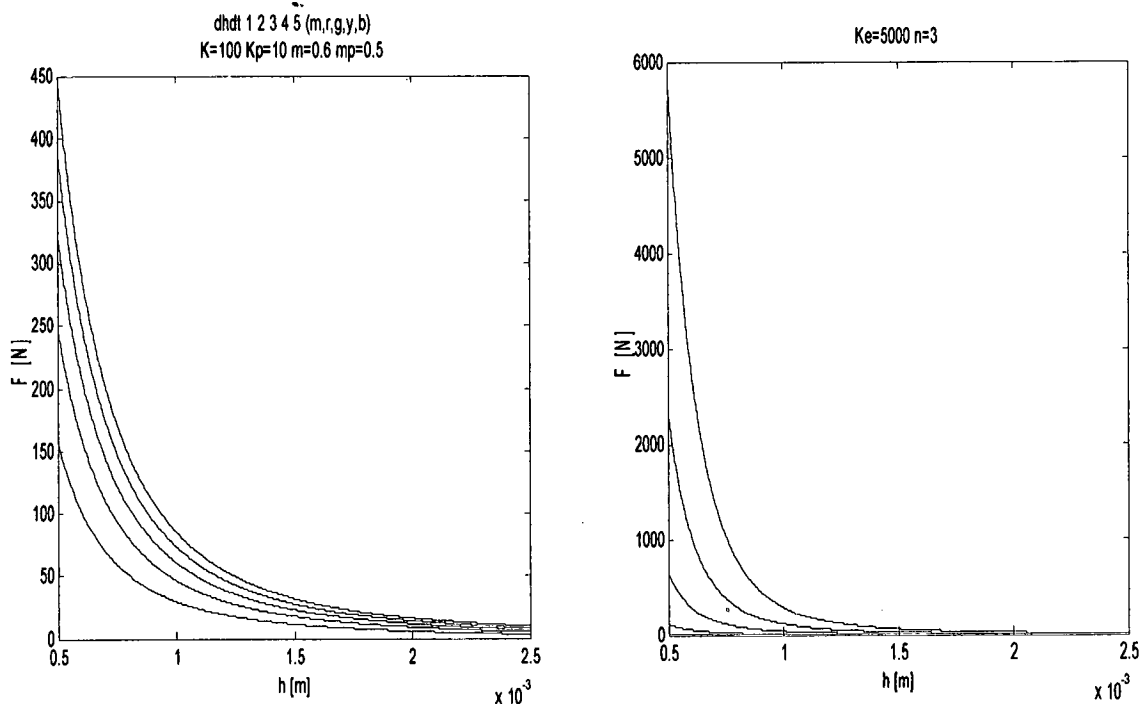


Fig.3 Force F for different velocities of discs (black 1 mm/s, red 2 mm/s...)

Fig.3 demonstrates the effect of speed dh/dt ; Force F increases with increased velocity as expected. Fig. 4 shows the effect of wall slip: lubricated walls ($\delta \rightarrow 0.5$) exhibit smallest forces (also as expected). Trends demonstrating compressibility are demonstrated in Fig. 5 that confirms correct behavior.

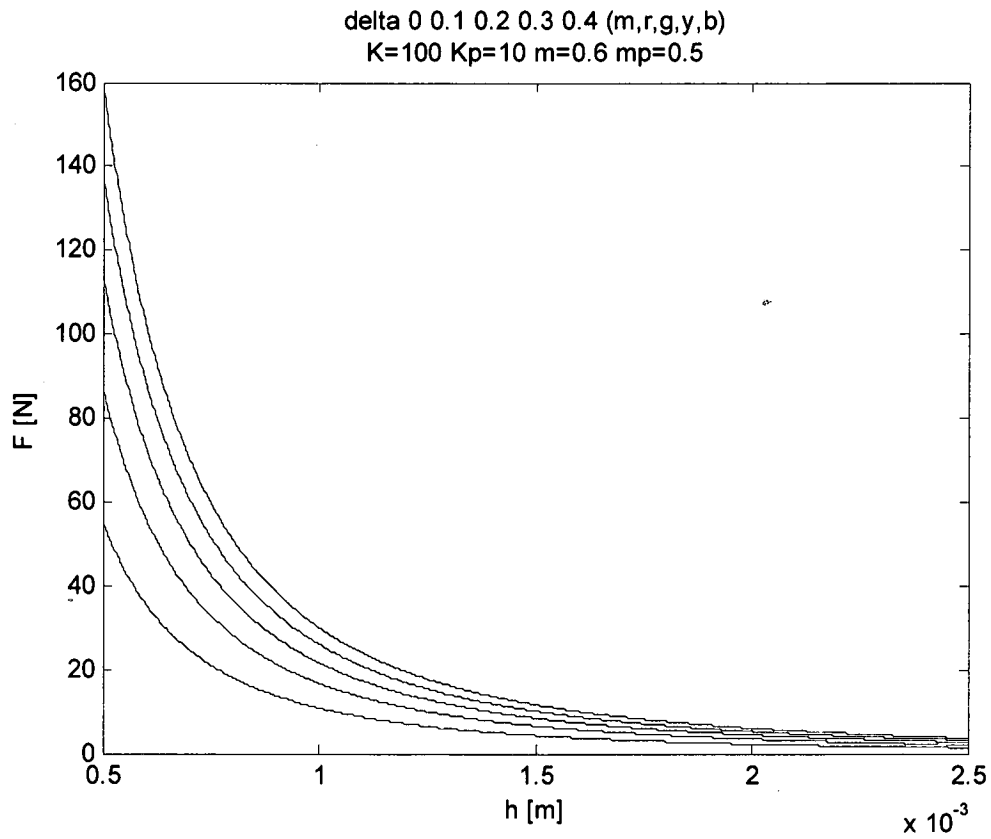


Fig.4 Force F for different slip velocities

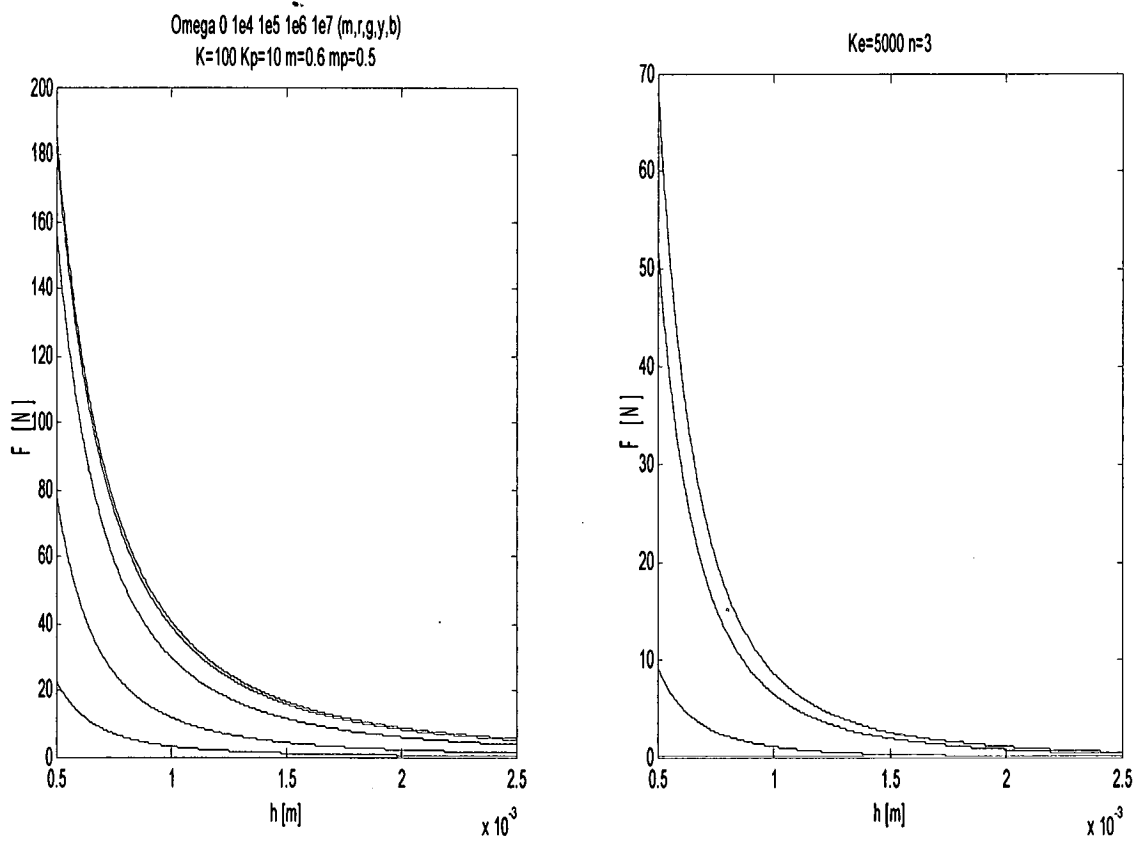


Fig.5 Force F for different mass fractions of air Ω (shear)

The simple purely ~~elastic~~ ^{elastoviscous} model having only 3 parameters (K_e , μ and Ω) was

Generally speaking the elongation model is more sensitive to the porosity of compressed material (Ω) and also to the velocity of discs. It is not possible to compare consistency coefficients K , K' and K_e because the first describe material response to shear deformation while the second response to stretching. The K_e/K can be interpreted as a Trouton ratio of elongation and shear viscosity. This ratio is 3 for Newtonian fluids but can be many times greater for non-Newtonian fluids.

also implemented in Matlab and comparison with shear dominated model in FITS-3 and

MATERIAL AND METHODS

Natural bovine collagen (type I) solution with a solid content of 7.1% was used for the experiment. Small sample pieces of weight about 60 grams convenient for experiments were separated from bulk collagen solution mass. Pieces were hand kneaded to obtain a homogeneous consistency. Thus prepared preformed sample was pressed into cylindrical hoops of diameter 55 mm. Expressed cylindrical sample was compacted to the height of 20 mm. This is the initial height of the sample for the squeezing flow experiment. The sample was then wrapped in plastic wrap to prevent drying. After creating the required number of samples for the experiment pooled samples were cyclically vacuumed to remove any air which could penetrate into the collagen solution mass during manual processing. The samples were then tempered in a thermostatic box at 10°C or 20°C.

Special instrument TA-XT2i (Stable Micro Systems, UK Ltd) was used for squeezing flow experiments. Squeezing flow experiment of collagen solution was made as compression of material between two metal plates. Upper plate had diameter 49 mm, bottom plate was the square on a side about 100 mm. Both plates were stuck Teflon foil to guarantee perfect slip during measurement. The collagen sample was placed on the lower plate and the upper plate was adjusted to a distance of 20 mm from the bottom plate. By this procedure there was set to the default position for measurement. After that position setting the measurement has been started, that causes compression between two plates at a constant speeds 1 or 3 or 5 mm per second. The experiment was finished until relative deformation reached value 0.95 (the relative deformation is calculated as 1-final thickness/initial thickness, therefore the value 0.95 corresponds to the final $h=0.5$ mm and initial thickness of sample 20 mm). The force acting on upper plate was continuously recorded at the upper plate position and time. Experiments were repeated four times for each speed of upper plate.

The actual mathematical modeling (nonlinear regression) was performed using software DataFit (Oakdale Engineering, USA). The main goal of the regression procedure was to find real values of parameters of the rheological model presented above.

RESULTS AND DISCUSSION

There are presented experimental data of force vs. actual distance of plates received during squeezing flow of collagen solution done at compression speeds 1, 3 and 5 mm/s and temperatures 10 and 20°C in figures 6 and 7. There is apparent specific variability probably caused by sample inhomogeneities generated by hand preparation of sample mass before the experiment. There is apparent decrease in force with decreasing speed compression.

Regression analysis ~~of recorder data~~ ^{of recorder data} by 6 parameter model (20+23) resulted to ~~unstable~~ physically unacceptable result. Optimization without constraints predicted for sample negative consistency coefficient.

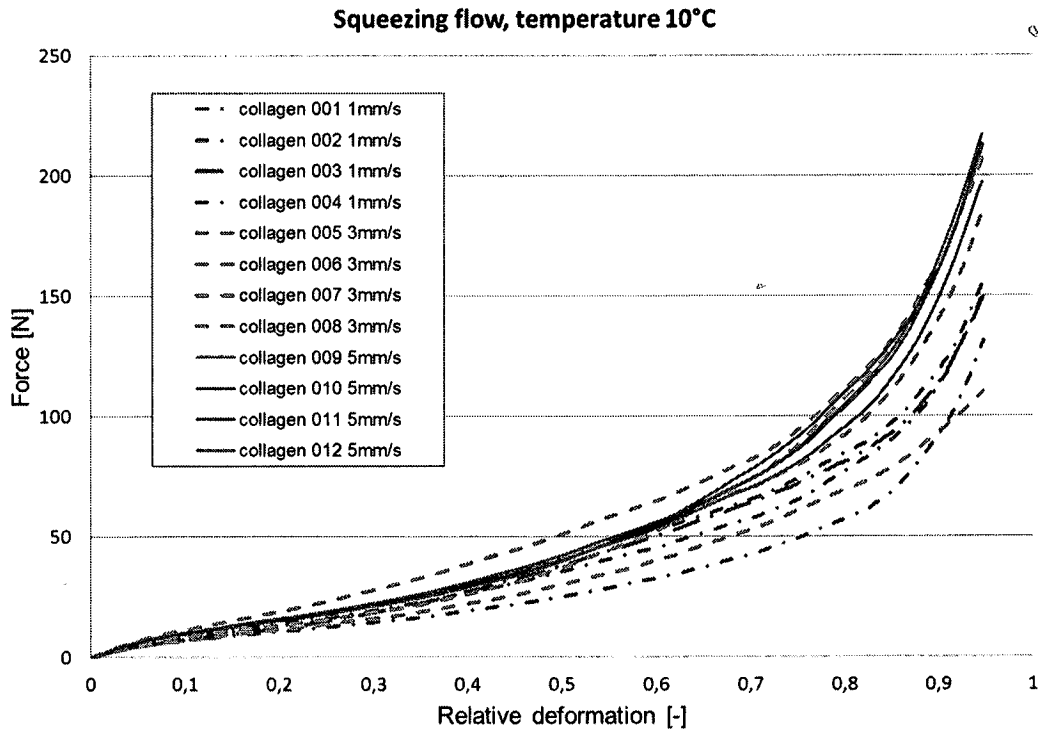


Fig. 6 Force vs. relative deformation of the sample, temperature 10 °C

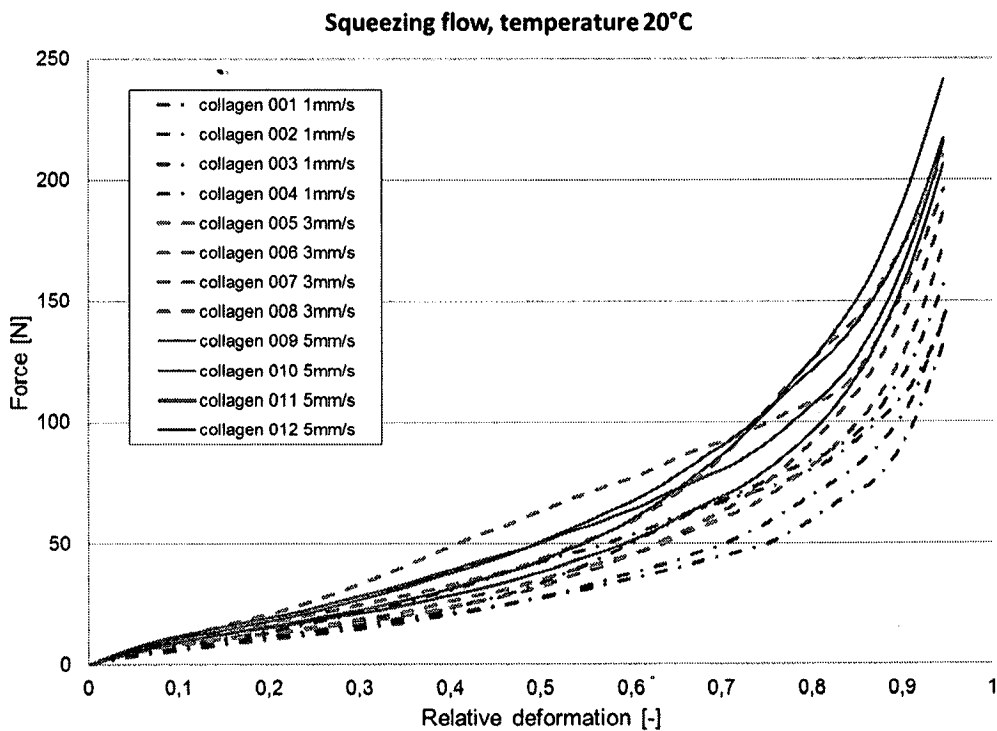


Fig. 7 Force vs. relative deformation of the sample, temperature 20 °C

The regression analysis was performed on all experimental data with aim to obtain parameters of the model of squeezing flow dominated with elongation. Values of parameters K_e , n and Ω were obtained. Two parametric regressions were also performed because of negative values of Ω assessed by three parametric. Then the parameter Ω were assumed equal to zero. The result of the regression is presented for one case of compression speed 3 mm per

seconds, for temperature 10 °C, in the figure 8. The parameters of the model are summarized in the table 2 for presented figure 8.

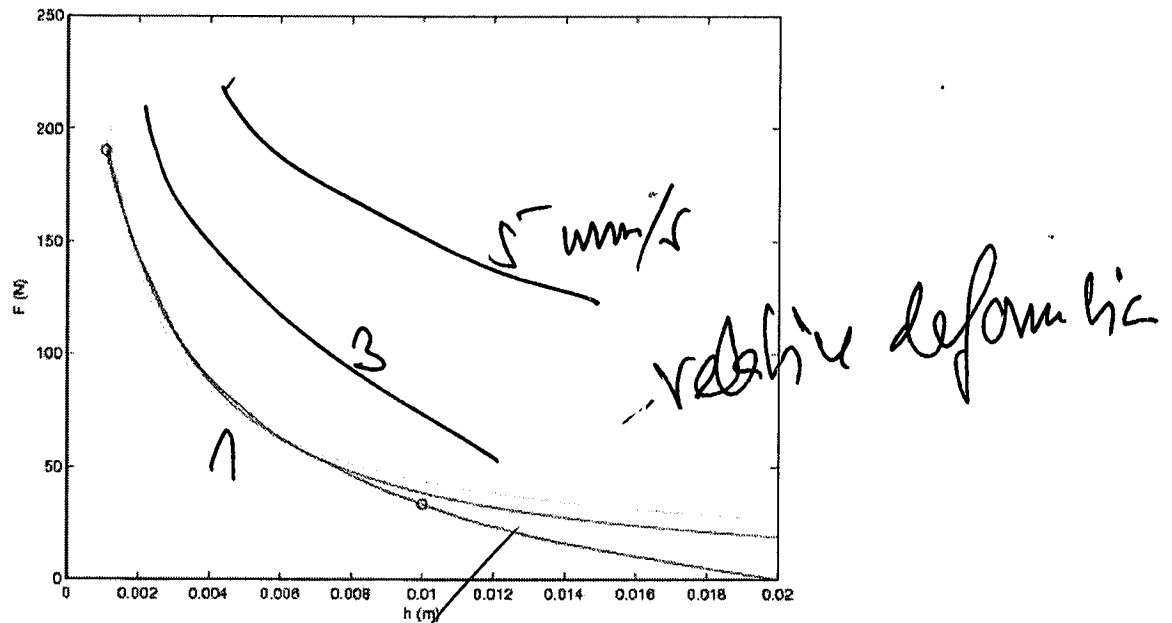


Fig. 8 Regression analysis of the experimental data, compression speed 3 mm per second, temperature 10°C, Force vs. height of the sample, blue – experiment, red – three parametric model, green – two parametric regression

Table 2 Parameters of the model presented in the figure 8.

Three parametric model (red line)			Two parametric model (green line)		
Ω	Ke	n	Ω	Ke	n
[Pa]	[Pa.s ⁿ]	[-]	[Pa]	[Pa.s ⁿ]	[-]
-97641	2519	1.118	0	140615	0.713

Diskuze, statistika?

CONCLUSIONS

The two models describing squeezing flow of compressible fluid were developed. The first model is based upon assumption that the compression force can be derived from the shear rate and from the transversal radial velocity profile. The model has 6 parameters: K , m is a power law characterization of rheogram, and K' , m' is a power law characterization of the first normal stress differences expressed again as a function of shear rate. The parameter Ω corresponds to the mass fraction of air in the tested sample. The last parameter δ is proportional to the slip velocity and must be less than 0.5. The shear rate is zero for the perfect slip $\delta=0.5$ and only the simple 3-parametric elongation model must be used. Both models predict expected trends: reduction of compression force with the increasing porosity and with the increasing slip at the surface of discs. Nevertheless the $F(h)$ curves are very similar and the model parameters are highly correlated. It is therefore rather difficult to distinguish the effect of compressibility, slip, normal stresses and pressure. It is desirable to carry out experiments at a broad range of velocities dh/dt and to evaluate as much as possible parameters from independent experiments.

Applied elongation model represented by equation (31) relatively well fit experimental data of compression force depending on distance between plates during squeezing flow of collagen solution. Influence of temperature and compression speed on parameters n and Ke had been statistically insignificant. This conclusion is valid only in ranges of tested parameters.

Surprising result is a failure of models based upon assumption of prevailing shear deformation - the best approximation of lubricated squeezing flow of concentrated collagen was obtained by a purely elongation model and by neglecting axial forces corresponding to shear deformation.

ACKNOWLEDGMENTS

This work was supported by the grant number 14-23482S of the Czech grant agency.

LIST OF PARAMETERS

A	coefficient in equations (1), (2), (3), (4), [N.m ^m , N.m ³]
B	coefficient in equations (1), (2), (4), [N.m, N.m ^{2q} , N.m ⁴]
F	force on plates during compression, [N]
g	exponent in equation, [-]
h	distance of plates, [m]
h'	reduced distance of plates, [m]
K	consistency coefficient – shear flow, [Pa.s ^m]
K'	consistency coefficient of the first difference of normal stresses, [Pa.s ^{m'}]
K _e	consistency coefficient – elongation flow, [Pa.s ⁿ]
M _g	mass of air, [kg]
M _l	mass of liquid, [kg]
<i>m</i>	mass flow rate of compressed material, [kg.s ⁻¹]
m	power-law index for shear, [-]
m'	power-law index for the first difference of normal stresses, [-]
N ₁	first normal stress difference, [Pa]
n	power-law index for elongation, [-]
p	pressure, [Pa]
p _a	atmospheric pressure, [Pa]
r	radial coordinate, [m]
R	upper plate radius, [m]
R _g	gas constant, [J.kmol ⁻¹ .K ⁻¹]
T	temperature, [K]
t	time, [s]
V	volume of sample, [m ³]
u _R	slip velocity, [m.s ⁻¹]
u _r	radial velocity, [m.s ⁻¹]
u _z	axial velocity, [m.s ⁻¹]
Z	compression rate, [Pa.m ^(-m-1)]
z	axial coordinate, [m]

Greek letters

ε _r	elongation rate in radial direction, [m ² .s ⁻¹]
ε _z	elongation rate in axial direction, [m ² .s ⁻¹]
ε _φ	elongation rate in tangential direction, [m ² .s ⁻¹]
φ	scale factor, [-]
δ	relative slip parameter, [-]
ρ _g	density of air, [kg.m ⁻³]
ρ _l	density of incompressible liquid, [kg.m ⁻³]
τ _{φφ}	normal stress in tangential direction, [Pa]
τ _{zr}	shear stress, [Pa]

τ_{rr}	normal stress in radial direction, [Pa]
τ_{zz}	normal stress in axial direction, [Pa]
Ω	porosity of compressed material, [-]
ω_g	relative mass fraction of the gas, [-]

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