PARALLEL FLOW ASYMMETRIES DUE TO NON-UNIFORM TEMPERATURES INVESTIGATED BY TRACER TECHNIQUES

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Abstract. The effect of flow asymmetry was observed experimentally in lateral parallel channels of continuous direct ohmic heater. While the flow in parallel channels is uniform at isothermal conditions, one stream may be delayed and even stopped or reversed if the temperature differences between channels are too high in case of heating. The effect can be explained by buoyancy and theoretical analysis predicts existence of two solutions, symmetric and asymmetric distribution of flowrates, which can be stable within a certain range of temperatures and flowrates. Integral model, based upon momentum and heat balances has been suggested with the aim to predict influence of heater geometry, fluid properties and operational parameters (flowrates, intensity of heating) upon the RTD of parallel laminar flows. The model takes into consideration also the cross-flow between the main parallel streams. This "zonal" model enables to calculate residence time distribution (RTD) of the heater and results predict that the effects of asymmetries upon RTD is significant. Therefore the measurement of RTD characteristics seems to be promising method for detection of the parallel flow asymmetries. Experimental verification was based upon a) flow visualisation (injection of a coloured tracer and monitoring the tracer by Canon MV-100 camera), b) measurement of temperature profiles (11 thermometers Pt100), and on c) stimulus response experiments using KCl as a tracer for conductivity methods (2 Pt conductivity probes) and Tc99 as a radioisotope tracer (collimated scintillation detectors). Results confirm predicted influence of operational parameters and geometry (diameter of channels) on the stability of flow.

INTRODUCTION

Parallel flows are typical for many important apparatuses of process industries, e.g. flows in shell&tube or plate heat exchangers, heaters, reactors. Sometimes flow irregularities, instabilities or just only non-uniform distribution of flow in parallel channels occur. These undesirable phenomena can be caused by natural convection if the apparatus operates at non-isothermal conditions, which is typical for heat exchangers or heaters.

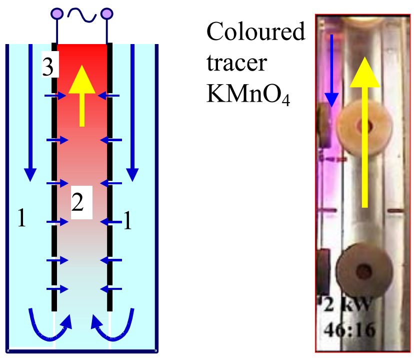

Figure 1. Continuous ohmic heater (scheme and photograph showing asymmetry of flows). 1-lateral channels;2-central channel,3-electrodes.

The effect of flow asymmetry was observed experimentally in lateral parallel channels of continuous direct ohmic heater, with two planar electrodes (electrical current flows directly through the heated liquid), see Fig.1. Liquid enters the top of heater and flows downwards through two rectangular channels where liquid is preheated only by warm electrodes. At the bottom of heater the two parallel streams join and liquid flows upward in a nearly uniform electrical field between electrodes (distance 0.036 m, voltage 220 V, 50 Hz). In order to suppress the electrode fouling, a perforation of electrodes was suggested, assuming that the cold cross-flow could displace overheated substance moving slowly along the electrode surface.

While the flow in parallel channels is uniform at isothermal conditions, a nonuniform distribution of flowrate and different temperature profiles exist in parallel channels at heating and also the cross flow is changed. These phenomena can be explained by the effect of buoyancy and can be detected by measuring of RTD.

STABILITY OF NON-ISOTHERMAL PARALLEL FLOWS

Parallel laminar flows in lateral channels of continuous ohmic heater are symmetric only at isothermal conditions. In case of heating, one stream is delayed and even stopped or reversed if the temperature increase is too high. This phenomenon can be explained by buoyancy.

A similar situation occurs in a simpler and probably more frequent case when two vertical parallel streams are separated by wall having a constant temperature T_e, see Fig.2.

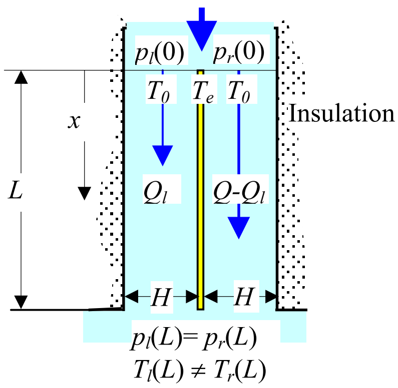

Figure 2. Parallel flows heated by wall at constant temperature T_e.

The analysis is based upon the fact that the pressure difference $p(L)$-$p(0)$ must be the same in the left and in the right channel at a steady state. We shall consider only two contributions to the pressure difference: the first represents viscous forces (written for flowrate Q_l [$m^3.s^{-1}$] in the left channel)

$$p_f(0) - p_f(L) = \frac{12\mu L f}{BH^3} Q_l \tag{1}$$

where B, H [m] are dimensions of rectangular cross-section (depth and width of channel respectively), and f equals 1 for laminar flow between plates or

$$f = \frac{1}{1 - \frac{192H}{\pi^5 B} \sum_{n=1,3,5,\ldots} \frac{1}{n^5} tgh \frac{n\pi B}{2H}} \tag{2}$$

for fully developed laminar flow in a rectangular channel B x H.

While the viscous component of pressure $p_f(x)$ decreases in the direction of flow, the hydrostatic pressure $p_b(x)$ increases. Assuming linear temperature dependence of density (coefficient proportionality β [K^{-1}]) and an exponential temperature profile $T_l(x)$,

$$\frac{T_l - T_e}{T_0 - T_e} = e^{-\frac{\alpha B}{\rho_0 c_p Q_l} x} \tag{3}$$

i.e. assuming a constant value of the heat transfer coefficient α [$W.m^{-2}.K^{-1}$] and heat capacity c_p [$J.kg^{-1}.K^{-1}$], the contribution of hydrostatic pressure can be expressed as

$$p_b(0) - p_b(L) = -\rho_0 g \int_0^L [1 - \beta(T_l - T_0)]dx = \rho_0 gL\{1 + \beta(T_0 - T_e)[1 - \frac{Q_l \rho_0 c_p}{\alpha BL}(1 - e^{-\frac{\alpha BL}{Q_l \rho_0 c_p}})]\} \tag{4}$$

This expression can be approximated by

$$p_b(0) - p_b(L) = -\rho_0 gL[1 + \beta(T_0 - T_e)\frac{\alpha BL}{2Q_l \rho_0 c_p}] \tag{5}$$

for high heat capacity of stream $Q\rho_0 c_p$ and for low value of heat transfer coefficient α.

Summing pressure differences corresponding to the friction forces (1) and buoayant forces (5) we can express equilibrium of forces in the left (lower index l) and right (index r) channel by equation

$$\frac{12\mu Lf}{BH^3} Q_l - \rho_0 gL[1 + \beta(T_0 - T_e)\frac{\alpha BL}{2Q_l \rho_0 c_p}] = \frac{12\mu Lf}{BH^3} Q_r - \rho_0 gL[1 + \beta(T_0 - T_e)\frac{\alpha BL}{2Q_r \rho_0 c_p}]. \tag{6}$$

It is obvious that this equation is always satisfied by symmetric solution $Q_l = Q_r$, nevertheless another, asymmetric, solution can exist too

$$Q_l Q_r = (T_e - T_0)\frac{g\alpha\beta B^2 H^3 L}{24\mu f c_p} \quad . \tag{7}$$

Eq.(7) can be rearranged to a dimensionless form by introducing the total flowrate Q and the ratio $\psi = Q_l/Q$

$$\psi(1 - \psi) = (T_e - T_0)\frac{g\alpha\beta B^2 H^3 L}{24\mu f c_p Q^2} = \mathbf{R} \quad . \tag{8}$$

The Eq.(8) predicts, that the asymmetric solution (i.e. distribution of flowrates satisfying balance of forces) can exist only if $\mathbf{R}$<1/4. For higher values of dimensionless parameter **R** the balance of forces cannot be satisfied giving rise flow reversals or other form of flow instabilities.

An important question appears whether and when the symmetric and asymmetric flow is stable. Let us assume a small disturbance of flowrate $Q_l+\delta Q$, $Q_r-\delta Q$, i.e. slightly increased flowrate in the left channel and decreased in the right channel. Then the pressure differences in the left and in the right channel are changed by increments (see Eq.(6))

$$\delta p_l = [\frac{12\mu Lf}{BH^3} - (T_e - T_0)\frac{g\alpha\beta BL^2}{2Q_l^2 c_p}]\delta Q, \qquad \delta p_r = [-\frac{12\mu Lf}{BH^3} + (T_e - T_0)\frac{g\alpha\beta BL^2}{2Q_r^2 c_p}]\delta Q \quad . \tag{9}$$

In the case that $\delta p_l > \delta p_l$ the pressure at the inlet to the left channel would be higher than the pressure in the right channel and this difference induces transversal flow towards the right channel. This redistribution of flow acts against the disturbance δQ, which means that the flowrates Q_l, Q_r will be stable if $\delta p_l - \delta p_l > 0$ for $\delta Q > 0$, i.e.

$$\frac{1}{\mathbf{R}} = \frac{24\mu f c_p Q^2}{g\alpha\beta LB^2 H^3 (T_e - T_0)} > \frac{Q^2}{2}(\frac{1}{Q_l^2} + \frac{1}{Q_r^2}) = \frac{1}{2}(\frac{1}{\psi^2} + \frac{1}{(1-\psi)^2}) \cdot \tag{10}$$

This inequality leads to the conclusion that the symmetric flow ($\psi=0.5$) will be stable if

$$\frac{1}{\mathbf{R}} > 4 \cdot \tag{11}$$

Example, corresponding to the geometry of lateral channels of the direct ohmic heater in Fig.1: B=0.08 m, H=0.008 m, L=0.56 m, flowrate of water Q=32 ml/s, ρ=1000 kg.m^{-3}, β=0.00093 K^{-1}, c_p=4200 J.kg^{-1}.K^{-1}, μ=1 mPa.s, estimated α=150 W.m^{-2}.K^{-1}. Temperature difference T_e-T_0 calculated from (11) using these parameters is approximately 10 ^{0}C, while flow asymmetries were observed within the range of heating power 1.8÷2.5 kW with corresponding water temperature increase 13÷18 ^{0}C in experiments. The deviation between prediction (11) and experiment could have been expected, because temperature of electrodes is not uniform and first of all is lower than the temperature of heated water.

The inequality (10) can be used for assessment of asymmetric solution (8) as well:

$$\frac{1}{\mathbf{R}} > \frac{1}{2}(\frac{1}{\psi^2} + \frac{1}{(1-\psi)^2}) = \frac{1-2\mathbf{R}}{2\mathbf{R}^2}, \qquad \mathbf{R} > \frac{1}{4} \tag{12}$$

In view of the fact that the asymmetric solution exists only for **R**<1/4, the stability requirement **R**>1/4 cannot be satisfied and asymmetric solution cannot be stable.

ZONAL MODEL OF PARALLEL AND CROSS-FLOWS

Continuous heater with perforated electrodes exhibits also rather complicated behaviour, because the cross flow through perforation is a decreasing function of heating power. The reason can be also found in buoyancy effects.

Geometry of two lateral channels (denoted by indices l,r) and central channel (c) is shown in Fig.3.

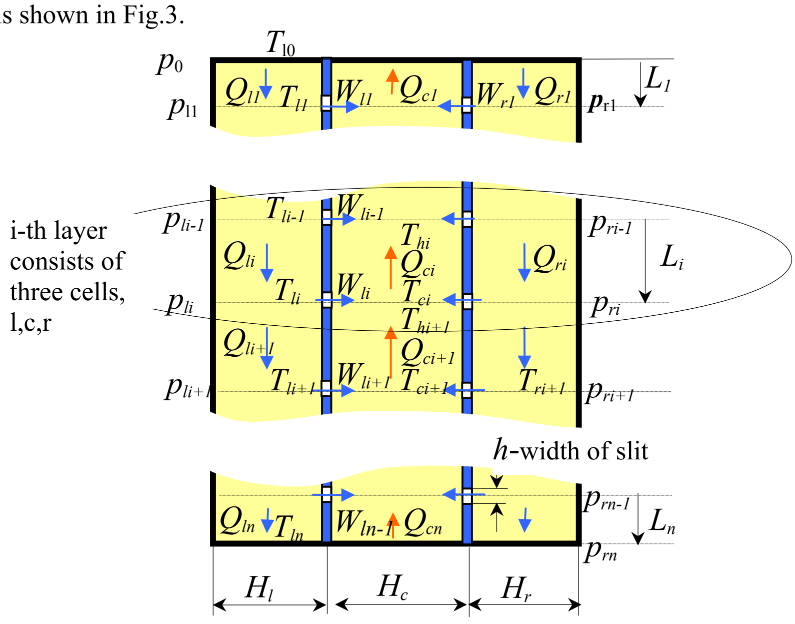

Figure.3.Zonal model

The following analysis is based upon integral balances of control volumes formed by sections of heater between slits in perforated electrodes. It is assumed, that the flow inside these volumes is fully developed, laminar and only in the axial direction.

Continuity equation can be written for each cell inside lateral channels

$$Q_{li} = Q_{l0} - \sum_{j=1}^{i-1} W_{lj} \qquad Q_{ri} = Q_{r0} - \sum_{j=1}^{i-1} W_{rj} \qquad \text{i=1,2,...,n} \tag{13}$$

and for the central channel

$$Q_{ci} = Q_{li} + Q_{ri} = Q_{l0} + Q_{r0} - \sum_{j=1}^{i-1} (W_{lj} + W_{rj}) \qquad \text{i=1,2,...,n} \tag{14}$$

Heat balance enables to estimate temperatures within the cells. Axial temperature profiles in lateral channels are assumed continuous, while the axial temperature profile are characterised by steps corresponding to injection of colder liquid in the places where electrodes are perforated. The heater is assumed to be thermally insulated therefore the balances of enthalpy in lateral channels give (written only for the left channel as an example)

$$\rho c_p Q_{li} (T_{li} - T_{li-1}) = \alpha_l L_i B (T_{ci} - T_{li}) \tag{15}$$

The axial temperature profile in the central channel is described by the step change at the interface between control volumes (mixing with lateral streams, cross-flow)

$$Q_{ci}T_{ci} - Q_{ci+1}T_{hi+1} = W_{li}T_{li} + W_{ri}T_{ri} \tag{16}$$

while the temperature change inside the cell is given by

$$\rho c_p Q_{ci}(T_{hi} - T_{ci}) = L_i B[-\frac{\alpha_l}{2}(T_{ci} + T_{hi} - T_{li} - T_{li-1}) - \frac{\alpha_r}{2}(T_{ci} + T_{hi} - T_{ri} - T_{ri-1}) + \kappa_i \frac{\Delta U^2}{H_c}] \tag{17}$$

where the last term describes volumetric heat source (κ_i [$S.m^{-1}$] is electrical conductivity of liquid and ΔU [V] voltage).

Pressure distribution is determined by hydraulic losses and by gravity. Hydraulic losses and buoyancy can be calculated using the same method as previously, giving axial profiles of pressure in lateral channels (written again for the left channel only)

$$p_{li} - p_{li-1} = \rho_0 g L_i [1 - \beta(\frac{T_{li-1} + T_{li}}{2} - T_0)] - L_i f_l Q_{li} \qquad i=1,2,...n \tag{18}$$

and in the central channel

$$p_{ci} - p_{ci-1} = \rho_0 g L_i [1 - \beta(\frac{T_{ci} + T_{hi}}{2} - T_0)] + L_i f_c Q_{ci} \tag{19}$$

where $$f_i = \frac{12\mu f}{BH_i^3}, \qquad i = l, r, c \cdot \tag{20}$$

Combining continuity equation with Eqs.(18-19) we can eliminate internal pressures and express the pressure at an arbitrary cell in terms of pressure p_0 at inlet and volumetric flowrate Q_{l0} at inlet

$$p_{li} - p_0 = \sum_{j=1}^{i} \{\rho_0 g L_j [1 - \beta(\frac{T_{lj-1} + T_{lj}}{2} - T_0)] - L_j f_l (Q_{l0} - \sum_{k=1}^{j-1} W_{lk})\} \cdot \tag{21}$$

Mention the fact that not only the inlet pressures, but also the pressures at the bottom should be the same $p_{ln} = p_{rn}$, giving the following relationship between inlet flowrates in the left and in the right lateral channel Q_{l0}, Q_{r0}:

$$\frac{\rho_0 g \beta}{2} \sum_{j=1}^{n} L_j (T_{lj-1} + T_{lj} - T_{rj-1} - T_{rj}) = L(f_r Q_{r0} - f_j Q_{l0}) + \sum_{j=1}^{n} L_j \sum_{k=1}^{j-1} (f_l W_{lk} - f_r W_{rk}) \cdot \tag{22}$$

The crossflow is related to the pressure differences between lateral and central channels. When calculating hydraulic resistance the two components are considered: Pressure drop corresponding to viscous forces which is proportional to $f_w = 12\mu f/(Bh^3)$ and pressure drop corresponding to acceleration of liquid (Borda's losses):

$$p_{li} - p_{ci} = \frac{\rho W_{li}^2}{2(Bh)^2} + f_w W_{li} \tag{23}$$

This quadratic equation enables to express the flowrate as a function of pressure difference

$$W_{li} = \frac{(Bh)^2}{\rho}(-f_w + \sqrt{f_w^2 + \frac{2\rho(p_{li} - p_{ci})}{(Bh)^2}}) \tag{24}$$

Iterative solution can be as follows: Flowrates at inlet (or ratio of flowrates Q_{l0}/Q_{c0}) are selected as an initial approximation. Cross-flows W_{li} W_{ri} are also estimated, for example as zero. Flowrates in cells are calculated from continuity equation (13-14). Knowing flowrates the complete temperature profiles are calculated from (15-17). Pressures are calculated from Eq.(18-20) and (23), giving improved values of cross-flow. New ratio of flowrates at inlet is calculated from Eq.(22), and all previous steps are repeated several times.

This model forms a basis for development of spatially localised RTD (Residence Time Distribution) model. Each cell in Fig.3 is described by a subsystem of one to five ideally mixed tanks, and flow-rates at inlets to these subsystems (including cross-flows) are given by preceding analysis. Thus it is possible to describe the spatial distribution of concentration of tracer as a function of time. Initial conditions (concentration of trace inside the heater) is zero, and inlet concentration at entry to lateral channels can be an arbitrary function of time (delta function or experimentally determined stimulus function). Resulting system of ordinary differential equations is integrated numerically.

It is true that approximation of tracer dispersion in convective laminar flow by a model formed by ideally mixed tanks is rather crude and cannot properly describe details of flow near walls. However, the model is simple and comparison with experiments indicates that the errors of calculated residence times are acceptable.

RTD EXPERIMENTS AND RESULTS

Experiments were performed with different thickness of lateral channels (H_l=H_r= 18, 11 and 7.8 mm). Temperature at lateral channels was recorded by two Pt100 probes and optical fibre probes Nortech TP-21-M02 which prove to be a sensitive indicator of flow instabilities, however results have not completely evaluated yet. Visualisation using $KMnO_4$ as a colour tracer indicates that the asymmetric flow could exist within a certain range of operational parameters, see the following table:

H [mm]	Q [ml/s]	$\Delta T_{exper.}$ [K]	T_e-T_0 Eq.(11)	Flow pattern	Evaluation
8	76	20	58	symmetric, stable	agree
8	65	22	42	asymmetric	acceptable
8	39	31	15.3	unstable	agree
18	83	14	14	symmetric, stable	acceptable
18	72	17	10.3	asymmetric ?	Eq.(11) unstable
18	68	18	9.2	unstable	agree

Residence time distribution was identified by using KCl and Tc-99 as a tracer [4]. Results obtained with these tracers are very similar, however the radioisotope is a better tracer at heating because solution of Tc99 has no effect upon direct ohmic heating (this is not true for solution of KCl which increases power and temperature when passing between electrodes).

Selected results of RTD measured in the system with narrow and wide lateral channels with and without heating and simulation by presented model for different width of perforation *h* are shown in Figs. 4a, b, c and d.

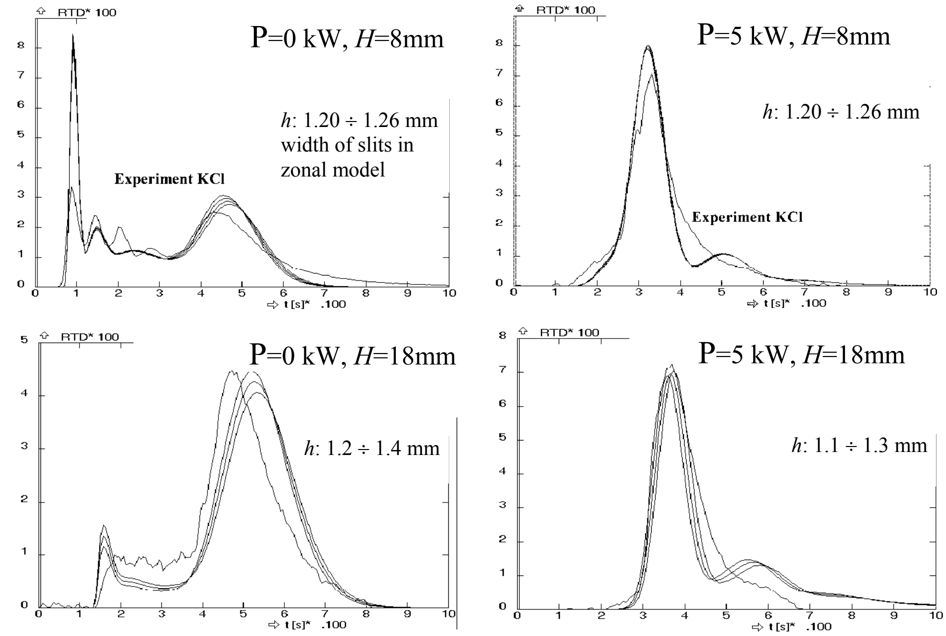

Figure 4. Experimental and simulated RTD for direct ohmic heater with perforated electrodes with different width h of perforation. Conductivity method.

CONCLUSION

Buoyancy has undesirable effects in heaters with downwards oriented parallel flows (non-uniform distribution and instability of flow), which can be suppressed by

- Decreasing width of channels (this is the most efficient way)
- Increasing viscosity
- Cross-flow (perforation of walls) has also positive effect improving stability.

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