FLOW OF PURELY VISCOUS NON-NEWTONIAN FLUIDS IN STRAIGHT NON-CIRCULAR DUCTS: A REVIEW AND COMPARISON OF PROCEDURES FOR RAPID ENGINEERING FRICTION FACTOR ESTIMATES

JIŘÍ ŠESTÁK, RUDOLF ŽITNÝ AND MARTIN DOSTÁL

Czech Technical University in Prague – Faculty of Mechanical Engineering, Institute of Process Engineering, Technická 4, 166 07 Prague 6, Czech Republic
sestak@fsid.cvut.cz

ABSTRACT

This paper presents a review and comparison of approximate methods for rapid estimates of the friction factor for stabilized flow of purely viscous fluids in non-circular ducts. Results of the approximate procedures, for the particular case of a power-law fluid, are compared with data obtained by numerical integration. For cross-sectional geometries formed by singly connected regions without sharp corners, all available approximate procedures yield results with relative deviations from the numerical data not exceeding about 5%. However, for cross sections formed by doubly connected regions such as for flow in the gap between a square duct with an inner centered cylindrical core, or for an eccentric annulus, deviations may exceed much more than 15% and neither of the existing methods can be recommended.

KEYWORDS: FLOW IN DUCTS, POWER-LAW FLUIDS, NON-CIRCULAR CROSS SECTIONS, FRICTION FACTOR

INTRODUCTION

For laminar flow of Newtonian fluids in ducts, theoretical as well as numerical results for the friction factor are most frequently expressed in term of the Poiseuille dimensionless group

$$Po = f\,\mathrm{Re}\,, \qquad (1)$$

where the Fanning friction factor f and Reynolds number are defined as,

$$f = \frac{D_e \Delta p}{2\rho \bar{u}^2 L}\,,\quad \mathrm{Re} = \frac{\bar{u} D_e \rho}{\mu}\,, \qquad (2,3)$$

where $D_e = 4A/O$ is the equivalent diameter, $\bar{u}$ denotes the volumetric mean velocity, Δp stands for the pressure difference on the duct length L and ρ, μ denote the fluid density and dynamic viscosity.

For Newtonian fluids, the $f\,\mathrm{Re}$ product in Eq. (1) depends upon the particular form of the cross-sectional geometry only, e.g. it's value is 16 for a circular tube. For power-law fluids for which the rheological model $\tau = k\dot{\gamma}^n$ holds, a generalized Reynolds number Re_B is defined,

$$\mathrm{Re}_B = \frac{\bar{u}^{2-n} D_e^n \rho}{2^{3(n-1)} k}\,, \qquad (4)$$

for which,

$$f\,\mathrm{Re}_B = f\,\mathrm{Re}_B\left(n, \text{cross}-\text{sectional geometry}\right). \qquad (5)$$

APPROXIMATE METHODS FOR CALCULATING THE FRICTION FACTOR

Kozicki, Chou and Tiu, [1] appear to be the first who proposed to estimate the friction factor from the equation

$$f\,\mathrm{Re}_B = 16\left[\frac{a + bn}{n}\right]^n, \qquad (6)$$

where a, and b are geometrical parameters which are determined from the corresponding solution for Newtonian flow in the same cross-sectional geometry, see. e.g. [1] or [2].

Miller, [3], proposed in 1972 the following approximation,

$$f\,\mathrm{Re}_B = 16\left[(a+b)\frac{3n+1}{4n}\right]^n. \qquad (7)$$

Since, according to Kozicki et al, [1],

$$a + b = f\,\mathrm{Re}/16\,, \qquad (8)$$

Miller's method employs only one parameter, more or less easily obtainable from the corresponding Newtonian solution.

In 1995, analyzing values of Kozicki's geometrical parameters, Delplace and Leuliet [4], deduced the following correlation between the parameters,

$$\frac{b}{a} = \frac{3}{a+b}\,. \qquad (9)$$

Substituting from Eq. (9) into Kozicki's Eq. (6) yields Delplace and Leuliet's approximation,

$$f\,\mathrm{Re}_B = 16\left[(a+b)\frac{3n+a+b}{(3+a+b)n}\right]^n, \qquad (10)$$

which again employs only one parameter, namely that in Eq. (8).

Recently, Liu and Masliyah [5], developed a three-shape-factor method embodied in the relation

$$f\,\mathrm{Re}_B = 16\left(k_1/2\right)^n\left(1 + \frac{1-n}{k_2 n}\right)^n k_3^{n-1}\,, \qquad (11)$$

similar to that developed by Kozicki et al, [1]. Since it is not difficult to show that the shape factors k_1 and k_2 are related to the a and b parameters by

$$k_1 = 2\left(a+b\right),\quad k_2 = \frac{b}{a} + 1\,, \qquad (12,13)$$

and taking in account that Liu and Masliyah adopted Deplace and Leuliet's assumption in Eq. (9), Eq. (11) is transformed into,

$$f\,\mathrm{Re}_B = 16\left[(a+b)\frac{3n+a+b}{(3+a+b)n}\right]^n k_3^{n-1}\,, \qquad (14)$$

which is very similar to Delplace and Leuliet's result in Eq. (10). In order to apply all the methods outlined so far, a Newtonian flow solution in the particular cross-sectional geometry is sufficient. A certain drawback of the method in Eq. (14) originates from the fact that the determination of the k_3 shape-factor requires a numerical solution for n different from unity.

A COMPARISON OF THE APPROXIMATE METHODS

Cross section formed by a singly connected region

In order to check the accuracy of all the four approximate methods for a cross-sectional geometry formed by a singly connected region, a symmetrical L-profile was used, see Figure. 1. Results obtained by a finite difference numerical technique, [6], were taken as reference "exact" values. For the particular case of $B/A = 0.5$, excellent agreement was found with values of $f\,\mathrm{Re}_B$ reported by Ta-Jo Liu, [7], from $n=1$ down to $n=0.5$.

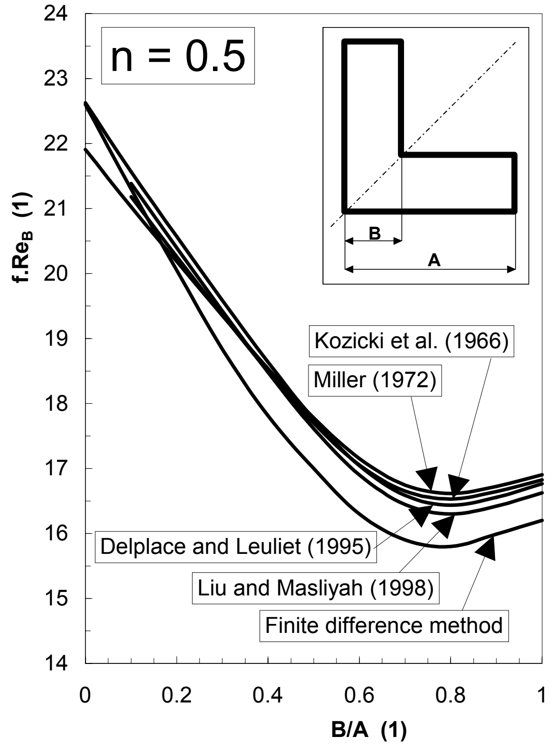

Figure 1. A comparison of the approximate methods for a symmetrical L-profile.

In Figure 2. the relative deviation ω,

$$\omega = \frac{\left(f\,\mathrm{Re}_B\right)_{approx} - \left(f\,\mathrm{Re}_B\right)_{numerical}}{\left(f\,\mathrm{Re}_B\right)_{numerical}}, \qquad (15)$$

is plotted against the B/A simplex. As expected, Liu and Masliyah's procedure yield the best results due to the fact that the k_2 and k_3 shape-factors were extracted from a numerical solution [7]. Kozicki et al. and Delplace and Leuliet's methods exhibit similar results, the latter being more advantageous due to the fact that it employs a single shape-factor only. Generally, all four methods predict correctly a minimum of $f\,\mathrm{Re}_B$ values near $B/A \doteq 0.75$, relative deviation of all methods remains below 5%.

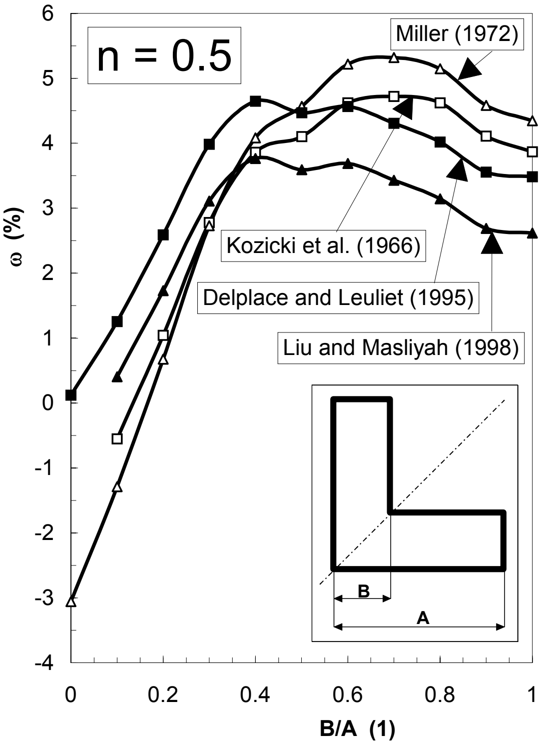

Figure 2. Relative error of approximate methods for a symmetrical L-profile.

Cross section formed by a multiply connected region

As an example of a cross section formed by a doubly connected region, the eccentric annulus geometry in Figure 3 was choosen. The shape factors a and b were determined from the Newtonian solution published by Piercy, Hooper and Winny [8]. Geometry of the cross section is defined by the gap curvature $\kappa = 2r/2R = d/D$ and by the dimensionless eccentricity $e^* = e/(R-r)$. Results of the approximate analyses were compared with numerical data reported by Guckes in 1975, [9], for $n = 0.5$. From Figure 3. and 4. it is clear that, for $e^* < 0.5$ the relative error of all three methods remains below 5%, Kozicki's method resulting perhaps in slightly more accurate values. Since k_3 values could not be obtained from the graphically presented data in reference [9], in view of the conclusions in the original work [5], the obvious choice is to set $k_3 \doteq 1$. If this is done, Liu and Masliyah's predictions coincides with Deplace's method, [4]. However, for values of $e^* > 0.5$, accuracy of all the approximations decreases rapidly and, with e^* approaching unity (i.e. for the inner core or tube touching the outer tube) makes all the approximate methods useless. A similar phenomenon has been reported for the doubly connected cross section between a circular core enclosed in a square duct, [10].

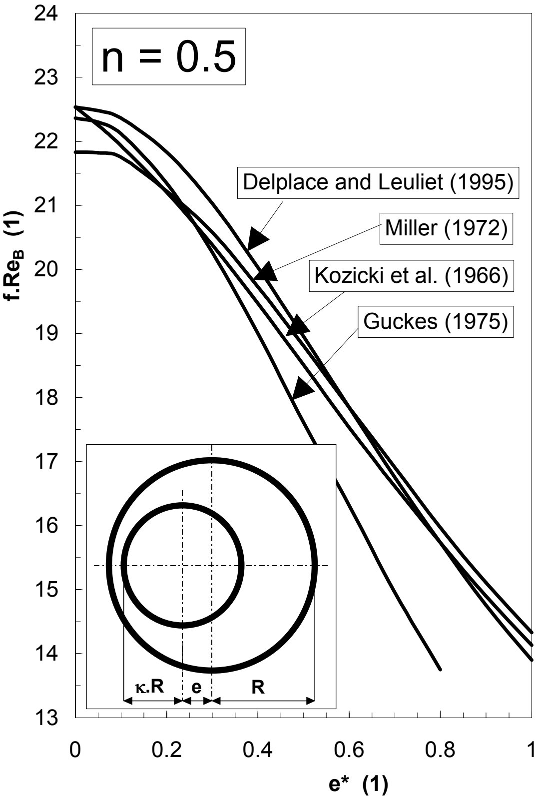

Figure 3. A comparison of approximate methods for an eccentric annulus.

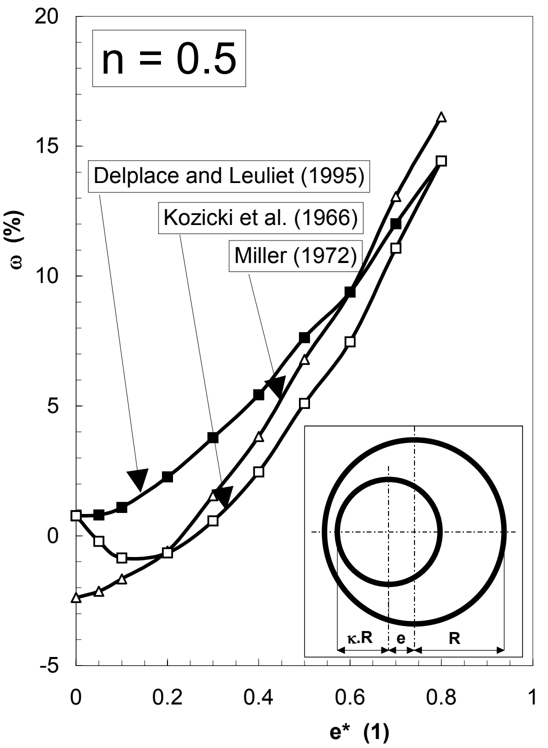

Figure 4. Relative error of approximate methods for an eccentric annulus.

CONCLUDING REMARKS

For cross sections formed by singly connected regions and values of $f\,\mathrm{Re}/16$ differing not much from unity, all four approximate procedures yield useful results with errors usually below 5%, mostly in the whole range of $0 < n < 1$. From the point of view of computational effort, Deplace and Leuliet's method may be recommended giving results with acceptable accuracy and just a single shape factor in Eq. (8). For cross sections formed by multiply connected regions, especially those exhibiting very sharp corners (such as in the gap of the annular geometry for $e^* \to 1$), neither of the methods available so far can be recommended and further work is needed.

ACKNOWLEDGEMENTS

This work was done as a part of a research project No.:101/99/1617 of the Grant Agency of the Czech Republic. Financial support of the Grant Agency is gratefully acknowledged.

Also, this work was partly sponsored by the Ministry of Education of the Czech Republic, Project No: J04/98: 212200008.

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