Algorithm for simultaneous calibration of several sensors (thermocouples, Pt100, pressure transducers).

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Problem description:

Let us assume that *M*-sensors detect the same quantity T^*_j at the j-th experiment (observation point). For example *M*-thermocouples submerged in a thermostat are at the same temperature T^*_{j} which is measured by a reference thermometer. Nevertheless, this thermometer is not ideal and its reading T_{j} is not precisely T^*_{j}. Output of sensors is voltage U_{ij} where i-is index of sensor (i=1,2,...*M*) and j- is index of observation point (j=1,2,...,*N* measurements at different temperatures of bath). The calibration procedure tries to find out coefficients of a relationship between output voltage and the value of measured quantity (temperature) for each sensor. The problem of calibration is how to take into account the information, that the temperature of bath is the same for all sensors. Their characteristics should not be therefore evaluated separately.

Procedure of simultaneous calibration:

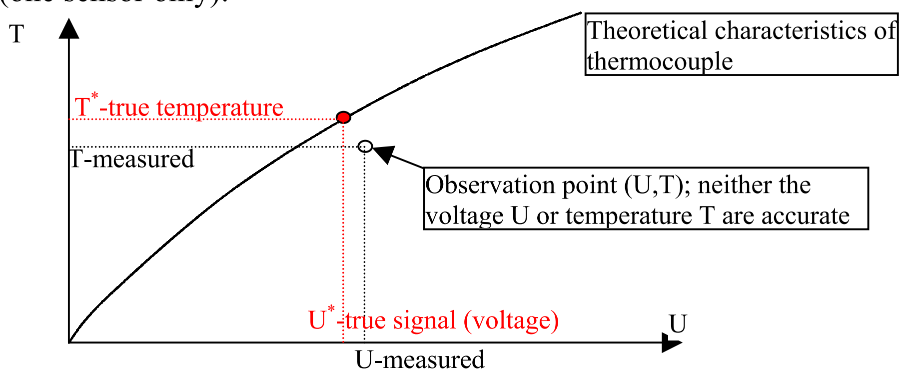
In the following we shall assume quadratic characteristics of sensors

$$T = a + bU + cU^2 \qquad (1)$$

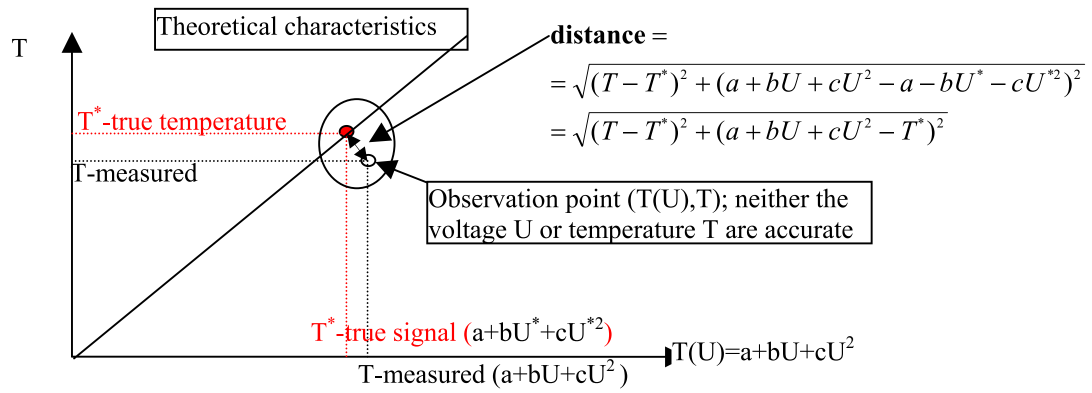
Variance of temperatures at j-th observation point can be expressed as

$$s_j^2 = w(T_j^* - T_j)^2 + \sum_{i=1}^{M} (T_j^* - a_i - b_i U_{ij} - c_i U_{ij}^2)^2 \qquad (2)$$

where w is a weight of accuracy of reference thermometer. If the thermometer is accurate the weight w can be large, because $T_j = T_j^*$. Explanation is based upon following figures for *M*=1 (one sensor only):



This figure can be transformed to the relationship between the temperature of bath and the temperature measured by sensor



The unknown temperature of bath T^* can be calculated so that the distance between the observed and true value is minimum. Then the true temperature follows from minimum of Eq.(2)

$$T_j^* = \frac{wT_j + \sum_{i=1}^{M}(a_i + b_i U_{ij} + c_i U_{ij}^2)}{w+M} \qquad (3)$$

Substituting this temperature back into expression of variance (2) and summing for all N-observation points we obtain

$$s^2 = \sum_{j=1}^{N}\{w[\frac{\sum_{i=1}^{M}(a_i + b_i U_{ij} + c_i U_{ij}^2) - MT_j}{w+M}]^2 + \sum_{i=1}^{M}[\frac{\sum_{k=1}^{M}(a_k + b_k U_{kj} + c_k U_{kj}^2) + wT_j}{w+M} - a_i - b_i U_{ij} - c_i U_{ij}^2]^2\} \qquad (4)$$

Unknown coefficients a_i, b_i, c_i can be calculated from the requirement of minimum of (4)

$$\frac{\partial s^2}{\partial a_l} = \frac{\partial s^2}{\partial b_l} = \frac{\partial s^2}{\partial c_l} = 0, \quad l = 1,2,\ldots,M$$

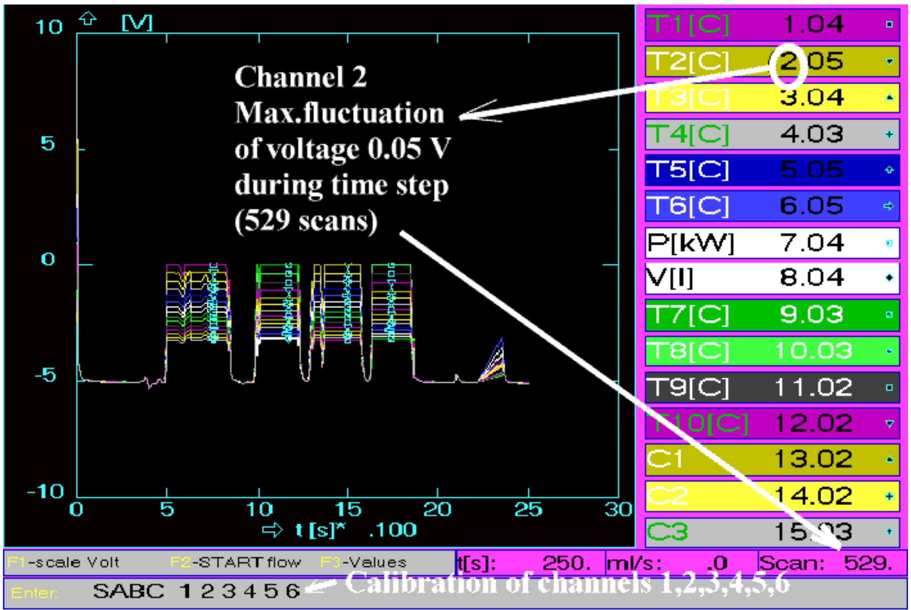
giving

$$(M+w)(a_l N + b_l \sum_{j=1}^{N} U_{lj} + c_l \sum_{j=1}^{N} U_{lj}^2) = w\sum_{j=1}^{N} T_j + \sum_{i=1}^{M}(a_i N + b_i \sum_{j=1}^{N} U_{ij} + c_i \sum_{j=1}^{N} U_{ij}^2) \qquad (5)$$

$$(M+w)(a_l \sum_{j=1}^{N} U_{lj} + b_l \sum_{j=1}^{N} U_{lj}^2 + c_l \sum_{j=1}^{N} U_{lj}^3) = w\sum_{j=1}^{N} T_j U_{lj} + \sum_{i=1}^{M}(a_i \sum_{j=1}^{N} U_{lj} + b_i \sum_{j=1}^{N} U_{ij} U_{lj} + c_i \sum_{j=1}^{N} U_{ij}^2 U_{lj}) \qquad (6)$$

$$(M+w)(a_l \sum_{j=1}^{N} U_{lj}^2 + b_l \sum_{j=1}^{N} U_{lj}^3 + c_l \sum_{j=1}^{N} U_{lj}^4) = w\sum_{j=1}^{N} T_j U_{lj}^2 + \sum_{i=1}^{M}(a_i \sum_{j=1}^{N} U_{lj}^2 + b_i \sum_{j=1}^{N} U_{ij} U_{lj}^2 + c_i \sum_{j=1}^{N} U_{ij}^2 U_{lj}^2) \qquad (7)$$

Equations (5),(6),(7) represent system of 3M linear algebraic equations for 3M unknown coefficients a_i, b_i, c_i . In the case that w>>M the system splits into M groups of 3 equations – this corresponds to the independent evaluations of a_i, b_i, c_i for each sensor and this approach is correct if the temperatures of bath are measured accurately.



Implementation:

Procedure of calibration is implemented in program PCL818. Directives for calibration are

SABC i1,i2,....
...
$Tref_l$
...
$Tref_N$
...
EABC w
The part of PCL818 source program

```
subroutine cal(icalib,tcalib,vcalib,mcal,ncal,w,a,b,c)
c
c Calibration of MCAL-thermocouples using NCAL-observation points
c
      dimension icalib(10),tcalib(50),vcalib(50,10),a(32),b(32),c(32)
      common al(10),bl(10),cl(10),am(3,3),bm(3)
c
c Initial approximation al,bl,cl=0
c
      al=0
      bl=0
      cl=0
      do iter=1,30
      do l=1,mcal
        u1=0
        u2=0
        u3=0
        u4=0
        bm=0
        do j=1,ncal
          u1=u1+vcalib(j,l)
          u2=u2+vcalib(j,l)**2
          u3=u3+vcalib(j,l)**3
          u4=u4+vcalib(j,l)**4
          bm(1)=bm(1)+w*tcalib(j)
          bm(2)=bm(2)+w*tcalib(j)*vcalib(j,l)
          bm(3)=bm(3)+w*tcalib(j)*vcalib(j,l)**2
        enddo
        am(1,1)=(mcal+w-1)*ncal
        am(1,2)=(mcal+w-1)*u1
        am(1,3)=(mcal+w-1)*u2
        am(2,1)=(mcal+w-1)*u1
        am(2,2)=(mcal+w-1)*u2
        am(2,3)=(mcal+w-1)*u3
        am(3,1)=(mcal+w-1)*u2
        am(3,2)=(mcal+w-1)*u3
        am(3,3)=(mcal+w-1)*u4
        do i=1,mcal
           if(i.ne.l)then
              u10=0
              u20=0
              u01=0
              u11=0
              u21=0
              u02=0
              u12=0
              u22=0
              do j=1,ncal
                u10=u10+vcalib(j,i)
                u20=u20+vcalib(j,i)**2
```

```
                u01=u01+vcalib(j,l)
                u11=u11+vcalib(j,i)*vcalib(j,l)
                u21=u21+vcalib(j,i)**2*vcalib(j,l)
                u02=u02+vcalib(j,l)**2
                u12=u12+vcalib(j,i)*vcalib(j,l)**2
                u22=u22+vcalib(j,i)**2*vcalib(j,l)**2
              enddo
              bm(1)=bm(1)+al(i)*ncal+bl(i)*u10+cl(i)*u20
              bm(2)=bm(2)+al(i)*u01+bl(i)*u11+cl(i)*u21
              bm(3)=bm(3)+al(i)*u02+bl(i)*u12+cl(i)*u22
            endif
        enddo
c solution 3 x 3 for l-th a,b,c
        call gelg(bm,am,3,1,1e-6,ier)
        al(l)=bm(1)
        bl(l)=bm(2)
        cl(l)=bm(3)
      enddo
      enddo
```

References:

Žitný R.: Experimentální metody II., Praha 1993 (only in electronic form)

Michálková Š.: Tok viskoelastické kapaliny ve výtlačném reometru, diploma work 92 242, CTU FME, 1992